NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2021

TECHNICAL MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

1.1 1.1.1 $m_{AC} = \dfrac{y_C - y_A}{x_C - x_A} = \dfrac{0-\sqrt{3}}{1-2} = \sqrt{3}$

1.1.2 $\tan\theta = m_{AC}$
$\tan\theta = \sqrt{3}$
$\theta = 60°$

1.1.3 $m_{BC} = \tan(\theta + 75°)$
$m_{BC} = \tan(60° + 75°)$
$= -1$

1.2 $-1 = \dfrac{y_C - y_B}{x_C - x_B} = \dfrac{0-2}{1-b}$
$-1 = \dfrac{-2}{1-b}$
$-1 + b = -2$
$b = -1$

1.3 $m_{BC} \times m = -1$
$-1 \times m = -1$
$m = 1$
$\text{midpoint}\left(\dfrac{1-1}{2}\,;\,\dfrac{0+2}{2}\right)$
$\text{midpoint}(0\,;1)$
$y = mx + c$
$1 = (1)(0) + c$
$1 = c$
$y = x + 1$

QUESTION 2

2.1 2.1.1

$$r^2 = x^2 + y^2$$
$$r^2 = (4)^2 + (3)$$
$$25 = x^2 + y^2$$

2.1.2

$$\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$
$$= \sqrt{\left(\frac{-16}{13} - 4\right)^2 + \left(\frac{-63}{13} - 3\right)^2}$$
$$= \frac{34\sqrt{13}}{13} = 9{,}43$$

2.1.3 Gradient of radius = $\frac{3}{4}$

Gradient of tangent = $\frac{-4}{3}$

Option 1

$$\therefore y = mx + c$$
$$\therefore 3 = \frac{-4}{3}(4) + c$$
$$\therefore \frac{25}{3} = c$$
$$\therefore y = \frac{-4}{3}x + \frac{25}{3}$$

Option 2

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{-4}{3}(x - 4)$$
$$y - 3 = \frac{-4}{3}x + \frac{16}{3}$$
$$\therefore y = \frac{-4}{3}x + \frac{25}{3}$$

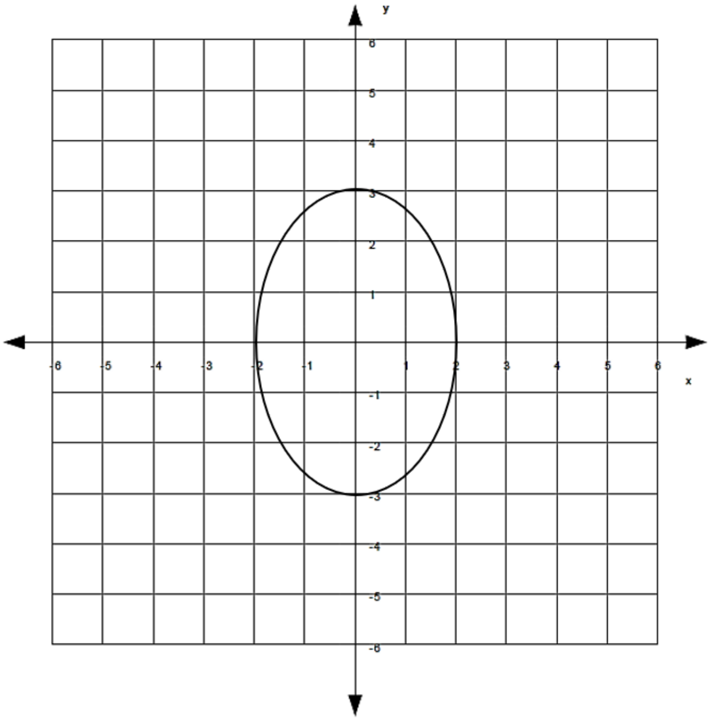
2.1.4

$$y = mx + c$$
$$\therefore y = \frac{-4}{3}x + 5$$
$$\therefore 0 = \frac{-4}{3}x + 5$$
$$\therefore x = \frac{15}{4}$$

2.2 $9x^2 + 4y^2 = 36$

$$\frac{9x^2}{36} + \frac{4y^2}{36} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$



x-intercepts
y-intercepts
shape

QUESTION 3

3.1 3.1.1 **Option 1**

$$\theta = \frac{4\pi}{15} \times \frac{180°}{\pi} = 48°$$

$$= \frac{\sec^2(62°) - 1}{\tan(48°)}$$

$$= \frac{\frac{1}{\cos^2(62°)} - 1}{\tan(48°)}$$

$$= \frac{\frac{1}{\cos^2(62°)} - 1}{\tan(48°)}$$

$$\approx 3{,}18$$

ALTERNATIVE

$$\beta = 62° \times \frac{\pi}{180°} = \frac{31\pi}{90}$$

$$\frac{sec^2\left(\frac{31\pi}{90}\right) - 1}{tan\left(\frac{4\pi}{15}\right)}$$

$$= \frac{\frac{1}{cos^2\left(\frac{31\pi}{90}\right)} - 1}{tan\left(\frac{4\pi}{15}\right)}$$

$$\approx 3{,}18$$

Option 2

$$\theta = \frac{4\pi}{15} \times \frac{180°}{\pi} = 48°$$

$$\frac{\sec^2\beta - 1}{\tan\theta}$$

$$= \frac{\tan^2\beta}{\tan\theta}$$

$$= \frac{\tan^2(62°)}{\tan(48°)}$$

$$\approx 3{,}18$$

3.1.2 $\tan 48° = \dfrac{4}{OC}$

$$OC = \frac{4}{\tan 48°} \quad \text{OR} \quad \frac{4}{tan\left(\frac{4\pi}{15}\right)}$$

$$\therefore OC = 3{,}6$$

3.1.3 $\tan 62° = \dfrac{AC}{OC}$

$$AC = 3{,}6 \tan 62°$$

$$\therefore AC \approx 6{,}77$$

$$\therefore AB = AC - BC$$

$$\therefore AB = 6{,}77 - 4 = 3{,}77$$

3.2 $$=\frac{3\sec^2(180^\circ-30^\circ)\cos 180^\circ}{\tan(360^\circ-45^\circ)-\cos^2(180^\circ+60^\circ)}$$

$$=\frac{\frac{3}{\cos^2(180^\circ-30^\circ)}(-1)}{-\tan 45^\circ-\cos^2 60^\circ}$$

$$=\frac{\frac{3}{\cos^2 30^\circ}(-1)}{-\tan 45^\circ-\cos^2 60^\circ}$$

$$=\frac{\frac{-3}{\left(\frac{\sqrt{3}}{2}\right)^2}}{-1-\left(\frac{1}{2}\right)^2}$$

$$=\frac{-3\times\frac{4}{3}}{-\frac{5}{4}}=\frac{16}{5}$$

3.3 $\sin 2x = 0{,}473$

Ref angle $=28{,}229^\circ$

$\therefore 2x = 28{,}23^\circ$ or $2x = 180^\circ - 28{,}23^\circ$

$\therefore x = 14{,}11^\circ$ or $x \approx 75{,}89^\circ$

3.4 LHS:

$$\frac{\cos x}{\sin x}+\frac{\sin x}{1+\cos x}$$

$$=\frac{\cos x(1+\cos x)+\sin x\ .\sin x}{\sin x(1+\cos x)}$$

$$=\frac{\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)}$$

$$=\frac{\cos x+1}{\sin x(1+\cos x)}$$

$$=\frac{1}{\sin x}$$

$\therefore$ LHS = RHS

3.5 $$\frac{\sin^2\theta(-\cot\theta)}{-\cos\theta}$$

$$=\frac{\sin^2\theta}{\cos\theta\tan\theta}$$

$$=\frac{\sin^2\theta}{\sin\theta}$$

$$=\sin\theta$$

QUESTION 4

4.1 4 amplitude for f
1 amplitude for g

4.2 $A(75,96° ; -0,97°)$

4.3 120°

4.4 4.4.1 $\therefore f(x) \geq g(x)$
$\therefore x \in [0° ; 75,96°]$ and $x \in [225,96° ; 360°]$

4.4.2 $x = 120° ; 300°$

QUESTION 5

5.1 $\sin 38° = \dfrac{EF}{5\ m}$ OR $\dfrac{EF}{\sin 38°} = \dfrac{5}{\sin 90°}$

$\therefore 5\ \sin 38° = EF$

$\therefore EF \approx 3,08\ m$

5.2 $\cos 61° = \dfrac{3,08\ m}{AF}$ OR $\dfrac{AF}{\sin 90°} = \dfrac{3,08}{\sin 29°}$

$\therefore AF = \dfrac{3,08}{\cos 61°}$ $AF = \dfrac{3,08}{\sin 29°}$

$\therefore AF \approx 6,35\ m$

5.3 $GF^2 = AG^2 + AF^2 - 2(AG)(AF)\cos\left(G\hat{A}F\right)$

$(5)^2 = (6,8)^2 + (6,35)^2 - 2(6,8)(6,35)\cos A$

$G\hat{A}F \approx 44,53°$

$Area = \dfrac{1}{2} AG\ .AF.\ \sin\left(G\hat{A}F\right)$

$Area = \dfrac{1}{2}(6,8)(6,35)\sin(44,53°)$

$Area \approx 15,14\ m^2$

QUESTION 6

6.1 6.1.1

$\hat{A} = 90°$	(angle is a semi-circle)
$\hat{O}_1 = 40°$	(Angle at centre = 2 × angle at circumference)
$\therefore \hat{B}_1 = 40°$	(Corresponding angles AB//FO)

$$90° + 40° + x = 180°$$
$$\therefore x = 50°$$

6.1.2

$\Delta ADB /// \Delta EDO$	(line through midpt || to 2nd side)
$\therefore \dfrac{OD}{BD} = \dfrac{EO}{AB}$	(Prop. theorem)
$\therefore \dfrac{OD \cdot AB}{BD} = EO$	

ALTERNATIVE MARK ALLOCATION

$\therefore \dfrac{OD}{BD} = \dfrac{EO}{AB}$	(Prop. theorem)
$\therefore \dfrac{OD \cdot AB}{BD} = EO$	

6.2 6.2.1 In $\Delta ADP \equiv \Delta EFO$

$\hat{D} = \hat{F} = 90°$	(angles in semi-circles)
$DP = FO$	(radii given equal)
$AP = EO$	(diameters given equal)
$\therefore \Delta ADP \equiv \Delta EFO$	(RHS)

6.2.2

$D\hat{P}A = F\hat{O}E$	$(\Delta ADP \equiv \Delta EFO)$
$\therefore DP \parallel MF$	(alternate angles equal)
and $AO = OP$	(radii)
$\therefore AM = MD$	(midpoint theorem)

6.2.3 ΔADP and ΔEFO

6.2.4

$OE = 4$ units	(given)
$OF = 2$ units	(radius)
$OE^2 = OF^2 + EF^2$	(pyth)
$4^2 = 2^2 + EF^2$	
$12 = EF^2$	
$\therefore EF = 2\sqrt{3}$	

6.3 6.3.1

$T\hat{A}G = 2T\hat{A}E = 34°$	(Given)
$\therefore \hat{O}_1 = 2T\hat{A}G$	(Angle at centre = 2 × angle at circumference)
$\therefore \hat{O}_1 = 2(34°) = 68°$	

6.3.2 $\hat{G}_2 = O\hat{B}G$ (angles at equal sides; radii)

$\hat{G}_2 + O\hat{B}G + 68° = 180°$ (int angles of triangle)

$\therefore \hat{G}_2 = O\hat{B}G = 56°$

6.3.3 $AT = TB$ (given)

$\therefore \hat{T}_2 = 90°$ (line from centre of circle to midpoint of chord)

$\hat{T}_2 + \hat{G}_2 + T\hat{B}G = 180°$ (interior angles of triangle)

$90° + 56° + T\hat{B}G = 180°$

$T\hat{B}G = 34°$

$T\hat{B}G = \hat{A}_5 = 34°$ (tan-chord theorem)

6.3.4 $K\hat{A}B = K\hat{A}G + T\hat{A}G$

$\therefore K\hat{A}B = 34° + 34° = 68°$ and $K\hat{A}B = A\hat{D}B$ (tan-chord theorem)

$\therefore A\hat{D}B = 68°$

6.4 $BC = CD = 2$ units (Radius ⊥ chord)

$OD^2 = CD^2 + OC^2$ (Pyth)

$r^2 = 2^2 + (r-1)^2$

$r^2 = 4 + r^2 - 2r + 1$

$2r = 5$

$r = 2{,}5$ units

QUESTION 7

7.1 $v = \pi Dn$

$v = \pi(0{,}24 \text{ m})(5{,}31 \text{ rev/s})$

$v = 4$ m/s

7.2 $v = \pi Dn$

$\frac{4\ m}{s} = \pi(0{,}48\ m)n$

$n = 2{,}65$ rev/s

7.3 $\omega = 2\pi n$

$\omega = 2\pi(2{,}65)$

$\omega = 16{,}65$ rad/s

QUESTION 8

8.1 Volume $= \pi r^2 \times h$

$= \pi(1{,}75\text{ m})^2 \times (6{,}25\text{ m})$

$= 60{,}132\text{ m}^3$

$\approx 60\text{ m}^3$

8.2 8.2.1 $s = r\theta$

$s = (1{,}75\text{ m})(120° \times \frac{\pi}{180°})$

$s = 3{,}67\text{ m}$

8.2.2 Area $= \frac{rs}{2}$ **OR** Area $= \frac{r^2\theta}{2}$

$= \frac{(1{,}75\text{ m})(3{,}67)}{2}$

$= 3{,}21\text{ m}^2$

OR

$= \frac{(1{,}75\text{ m})^2\left(120° \times \frac{\pi}{180°}\right)}{2}$

$= 3{,}21\text{ m}^2$

8.2.3 Area = Sector area – Triangle area

$= 3{,}21\text{ m}^2 - 1{,}326\text{ m}^2$

$= 1{,}88\text{ m}^2$

8.2.4 Volume of diesel $= 1{,}88\text{ m}^2 \times 6{,}25\text{ m}$

$= 11{,}75\text{ m}^3$

Percentage filled $= \frac{\text{Filled volume}}{\text{Total volume}} \times 100\%$

$= \frac{11{,}75}{60} \times 100\%$

$\approx 19{,}58\%$

8.2.5 $4h^2 - 4dh + x^2 = 0$

$4(0{,}5)^2 - 4(3{,}5)(0{,}5) + x^2 = 0$

$\therefore x^2 = 6$

$\therefore x = \sqrt{6} \approx 2{,}45\text{ m}$

Area = l × b

= 2,45 × 6,25

$\approx 15{,}31\text{ m}^2$

QUESTION 9

9.1 **Option 1**

$$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + \ldots + o_{n-1}\right)$$
$$A_T = 2\left(\frac{0,45 + 0,21}{2} + 0,62 + 0,48 + 0,32 + 0,46 + 0,64 + 0,47\right)$$
$$A_T = 2(3,32)$$
$$A_T = 6,64\ \text{km}^2$$

Option 2

$$A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$$
$$A_T = 2(\frac{0,45+0,62}{2} + \frac{0,62+0,48}{2} + \frac{0,48+0,32}{2} + \frac{0,32+0,46}{2} + \frac{0,46+0,64}{2} + \frac{0,64+0,47}{2} + \frac{0,47+0,21}{2})$$
$$A_T = 2(3,32)$$
$$A_T = 6,64\ km^2$$

9.2 R5 250 000 income ÷ R40 per bale = 131 250 bales
hectares needed = 131 250 ÷ 350 = 375

Total area = 375 × 0,01 km^2 = 3,75 km^2

Total: 150 marks