NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2020

TECHNICAL MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

1.1 $m_{AB} = \dfrac{y_B - y_A}{x_B - x_A}$

$mm_{AB} = \dfrac{17-5}{12-3} = \dfrac{4}{3}$ substitute into correct formula

simplification

1.2 $m_{AB} \times m_{BC} = -1$

$\dfrac{4}{3} \times m_{BC} = -1$

$m_{BC} = \dfrac{-3}{4}$ simplification

$m_{BC} = \dfrac{y_B - y_C}{x_B - x_C} = \dfrac{-3}{4}$

$\dfrac{17-20}{12-k} = \dfrac{-3}{4}$ substitute into correct formula

$4(17-20) = -3(12-k)$ simplify

$-12 = -3(12-k)$

$4 = 12-k$

$k = 8$ simplification

1.3 $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$= \sqrt{(12-3)^2 + (17-5)^2}$ substitute into correct formula

$= \sqrt{225}$

$= 15$ units answer

$AB^2 + BC^2 = AC^2$ (Pyth)

$(15)^2 + (5)^2 = AC^2$ substitute into correct formula

$AC^2 = 250$

$AC = 5\sqrt{10}$ simplification

Perimeter $= 15 + 5 + 5\sqrt{10} = 20 + 5\sqrt{10}$ answer

QUESTION 2

2.1 2.1.1 $r^2 = x^2 + y^2$

$r^2 = 8^2 + 4^2$

$r^2 = 80$

$\therefore x^2 + y^2 = 80$

2.1.2 A(–8; –4)

2.1.3 A(–8; –4) B(8; 4)

$m_{AB} = \dfrac{y_A - y_B}{x_A - x_B} = \dfrac{-4-4}{-8-8} = \dfrac{1}{2}$

$\therefore y = \dfrac{1}{2}x$

2.1.4 $m_{PT} \times m_{AQ} = -1$ (tan $\perp$ rad)

$\therefore m_{PT} = -2$

$tan\theta = -2$

$\theta = 180° - 63,43° = 116,57°$

2.1.5 $y = -2x + c$

$-4 = -2(-8) + c$

$c = -20$

$\therefore y = -2x - 20$

OR

$y - y_1 = m(x - x_1)$

$y + 4 = -2(x - 8)$

$y = -2x - 20$

2.1.6 $y = -2x + -20$

$-10 = -2t + -20$

$t = -5$

OR

m(AT) = –2

$\dfrac{-4+10}{-8-t} = -2$

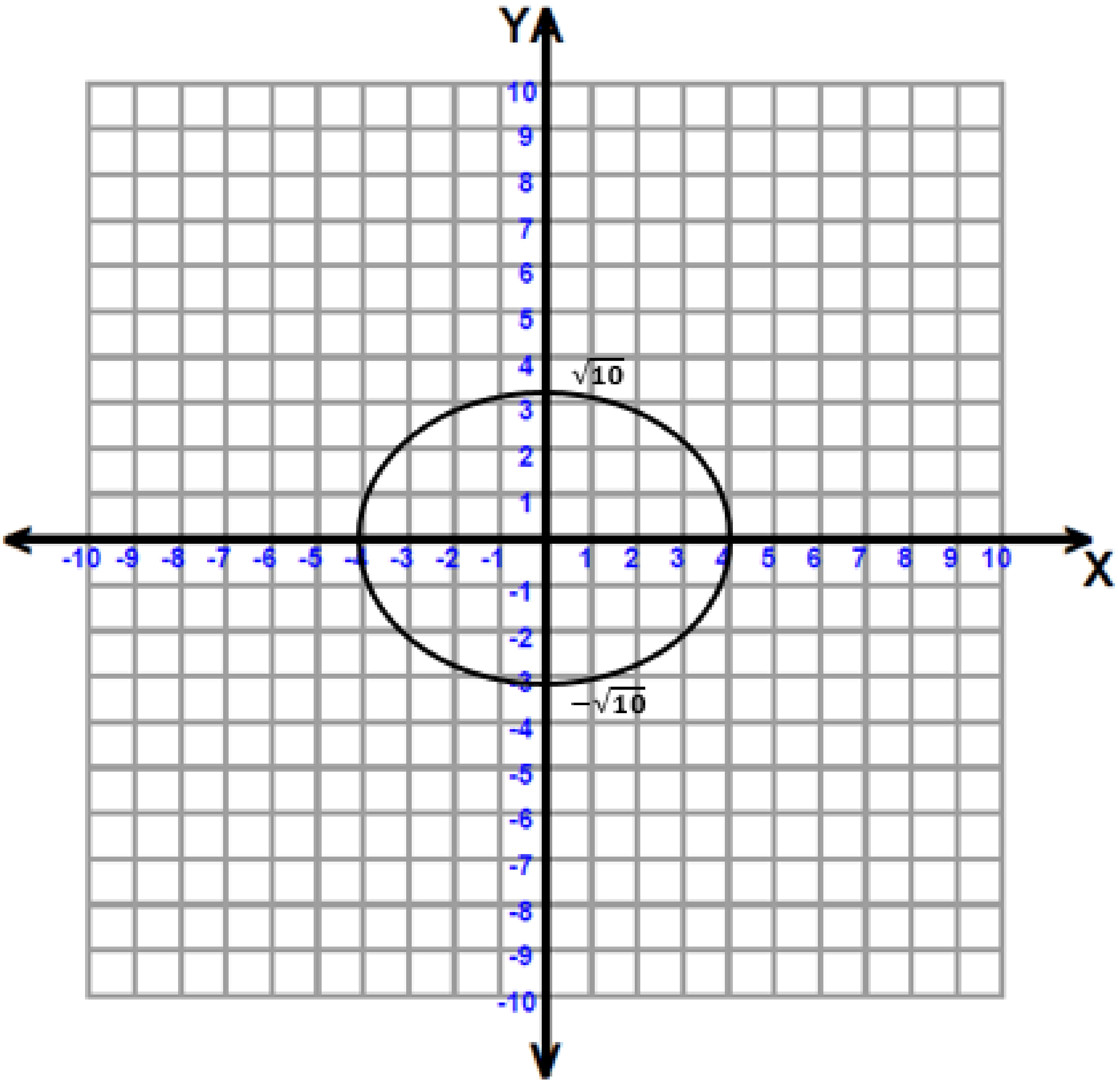
16 + 2t = 6

2t = –10

t = –5

2.2 Sketch the graph defined by $\frac{x2}{16}+\frac{y2}{10}=1$.

Clearly show ALL the intercepts with the axes.



x-intercepts at 4 and -4

y-intercepts at $\sqrt{10}$ and $-\sqrt{10}$

shape

QUESTION 3

3.1 3.1.1 $= 6\left(\dfrac{4}{2\sqrt{13}}\right) - 3\left(\dfrac{4}{6}\right)$

$= \dfrac{-26 + 12\sqrt{13}}{13}$

$x^2 + y^2 = r^2$

$6^2 + 4^2 = AO^2$

$52 = AO^2$

$\therefore\ AO = 2\sqrt{13}$

3.1.2 $= \left(\dfrac{2\sqrt{13}}{4}\right)^2$

$= \dfrac{13}{16}$

3.2 3.2.1 $= \text{cosec}\left(\dfrac{\pi}{3} - \dfrac{\pi}{6}\right)$

$= \text{cosec}\left(\dfrac{\pi}{6}\right)$

$= \dfrac{1}{\sin\left(\dfrac{\pi}{6}\right)}$

$= 2$

3.2.2 $= 2\cot\left(\dfrac{\pi}{6}\right)$

$= \dfrac{2}{tan\,\dfrac{\pi}{6}}$

$= \dfrac{2}{\dfrac{1}{\sqrt{3}}}$

$= 2\sqrt{3}$ OR $\approx 3{,}5$

3.3 3.3.1 $124{,}66^\circ \times \dfrac{\pi}{180^\circ} \approx 2{,}18$ radians

$57{,}46^\circ \times \dfrac{\pi}{180^\circ} \approx 1$ radian

3.3.2 $sec\left(2{,}18+1\right)$

$= sec\left(3{,}18\right)$

$= \dfrac{1}{cos\left(3{,}18\right)}$

$\approx -1{,}0$

3.4 $\tan\left(180^\circ-\alpha\right).\cos\alpha.\sin\left(180^\circ+\alpha\right)+cos^2\left(360^\circ+\alpha\right)$

$=(-\tan\alpha)\ \left(cos\alpha\right)\ (-\sin\alpha)\ \ +cos^2\alpha$

$=(-\sin\alpha)(-\sin\alpha)+cos^2\alpha$

$=\sin^2\alpha+cos^2\alpha$

$=1$

3.5 $\tan\left(x-15^\circ\right)=-1$

$\therefore\left(x-15^\circ\right)\ =180^\circ-45^\circ$ [reference angle (x – 15°)]

$\therefore x=150^\circ$

3.6 LHS: $sin^2A+tan^2A+cos^2A$ RHS: $\sec^2$A

$=1\ +tan^2A$

$=sec^2A$

$=$RHS

3.7 3.7.1 a = 3; b = 1; c = 0; d = 2

3.7.2 Periodicity = $\dfrac{360^\circ}{3}$

Periodicity = 120°

QUESTION 4

4.1 4.1.1 $\frac{30\text{m}}{\sin 65°} = \frac{\text{AB}}{\sin 50°}$

$$\frac{30\sin 50°}{\sin 65°} = \text{AB}$$

$$\text{AB} \approx 25{,}36 \text{ m}$$

4.1.2 $cos 25° = \frac{AD}{AB}$

$$\cos 25° = \frac{\text{AD}}{25.36\text{m}}$$

$$25{,}36 \times \cos 25° = \text{AD}$$

$$AD \approx 22{,}98 \text{ m}$$

OR

$$\frac{AD}{\sin 65°} = \frac{25{,}36}{\sin 90°}$$

$$AD = \frac{25{,}36 \times \sin 65°}{\sin 90°}$$

$$AD \approx 22{,}98 \text{ m}$$

4.2 4.2.1 $AC^2 = AB^2 + CB^2 - 2AB.CB.cos 60°$

$$AC^2 = (680\ m)^2 + (420\ m)^2 - 2(680\ m)(420\ m)cos 60°$$

$$AC^2 = 353200$$

$$\text{AC} \approx 594 \text{ m}$$

4.2.2 120°

4.2.3 $\frac{AD}{\sin 41°} = \frac{594\ m}{\sin 120°}$

$$\therefore AD = \frac{594 \times \sin 41°}{\sin 120°}$$

$$\text{AD} \approx 450 \text{ m}$$

QUESTION 5

5.1 In $\Delta ABC: AC^2 = AB^2 + BC^2 \text{(Pyth)}$

$$AC^2 = (230)^2 + (230)^2$$

AC = $230\sqrt{2}$ **OR** $\therefore AC = 325{,}269\ldots$

$\therefore MC = \dfrac{230\sqrt{2}}{2}$ **OR** $\therefore MC = \dfrac{325{,}269}{2}$

$\therefore MC = 115\sqrt{2}$ **OR** $\therefore MC = 162{,}63$

5.2 $\cos M\hat{C}T = \dfrac{MC}{TC}$

$$\cos M\hat{C}T = \frac{115\sqrt{2}\text{ m}}{218\text{ m}}$$

$$\therefore \cos M\hat{C}T = 0{,}746$$

$$\therefore M\hat{C}T \approx 41{,}75^\circ$$

QUESTION 6

6.1 6.1.1 $\hat{H}_1 = \hat{F}_1 = 23°$ (angles in same segment)

$\hat{F}_1 + \hat{F}_2 = \hat{E}_3 = 50°$ (equal angles opp equal sides; radii)

$\therefore 23° + a = 50°$

$a = 27°$

6.1.2 $\hat{E}_1 = \hat{F}_2 + \hat{F}_3$ (angles in same segment)

$49° = 27° + b$

$22° = b$

6.2 6.2.1 $\hat{B}_2 + \hat{F} = 180°$ (opp. angles of cyclic quad)

$\hat{B}_2 + 35° = 180°$

$\therefore \hat{B}_2 = 145°$

6.2.2 $\hat{E}_1 = 2 \times \hat{F}$ (angle at centre = 2 x angle at circumference)

$\hat{E}_1 = 2 \times 35° = 70°$

6.2.3 $\hat{E}_1 + \hat{C}_3 + \hat{D}_2 = 180°$ (int. angles of triangle CED)

$70° + \hat{C}_3 + \hat{D}_2 = 180°$

$\hat{C}_3 = \hat{D}_2$ (equal angles at equal sides; radii)

$\therefore \hat{C}_3 = \hat{D}_2 = 55°$

6.2.4 $2\hat{A} = \hat{B}_2$ (angle at centre = 2 x angle at circumference)

$2\hat{A} = 145°$

$\therefore \hat{A} = 72{,}5°$

6.2.5 $\hat{A} = \hat{C}_1 = 72{,}5°$ (equal angles at equal sides; radii)

$\hat{C}_1 + \hat{C}_2 = 90°$ (angle in semi-circle)

$\therefore \hat{C}_2 = 17{,}5°$

6.3 6.3.1 $\hat{A}_1 = \hat{C}_1$ (angles opposite equal sides, given AD = DC)

$\therefore\ \hat{A}_1 = 37{,}2°$

6.3.2 $\hat{A}_1 = \hat{C}$ (angles opposite equal sides, given AD = DC)

$\hat{A}_1 = \hat{B}_1$ = 37,2° (tan-chord theorem)

$\hat{B}_2 = 90° - 37{,}2°$ (angle in semi circle)

$\therefore\ \hat{B}_2 = 52{,}8°$

6.3.3 $A\hat{B}E = 90°$ (angle in semi-circle)

$AE^2 = BE^2 + AB^2$ (Pythagoras)

$(13)^2 = (3{,}5)^2 + AB^2$

$\therefore AB = 12{,}52$ cm

6.4 6.4.1 $\dfrac{PB}{BR} = \dfrac{PC}{CA} = \dfrac{1}{2}$ (Proportionality theorem, CB // AR)

$$\frac{PA}{PQ} = \frac{PC + CA}{PC + CA + AQ}$$

$$\frac{3}{8} = \frac{1+2}{1+2+5}$$

$$\therefore \frac{BD}{BQ} = \frac{CA}{CQ}$$

$$= \frac{2}{5}$$

6.4.2 $\dfrac{\text{Area of }\Delta\text{PRA}}{\text{Area of }\Delta\text{QRA}}$

$$= \frac{0,5 \times \text{PA} \times \text{height}}{0,5 \times \text{QA} \times \text{height}}$$

$$= \frac{\text{PA}}{\text{QA}}$$

$$= \frac{3}{5}$$

QUESTION 7

7.1 Area of a sector $= \dfrac{rs}{2} = \dfrac{r^2\theta}{2}$,

r = radius, s = arc length and θ = central angle in radians

$$\text{Area} = \frac{(9\text{ cm})^2\left(80° \times \dfrac{\pi}{180°}\right)}{2}$$

$$\text{Area} = 18\pi\text{ cm}^2$$

Area of triangle AOC $= \dfrac{1}{2} \times AO \times OC \times \sin 80°$

$$\therefore Area = \frac{1}{2} \times 9\ cm \times 9\ cm \times \sin 80°$$

$$\therefore Area = 39{,}884714\ cm^2$$

Area of shaded segment $= 18\pi\ cm^2 - 39{,}884714\ cm^2$

$$\approx 16{,}7\text{ cm}^2$$

7.2 7.2.1 $v = \pi Dn$

$$v = \pi(0{,}15)(4{,}2)$$

$$v = 1{,}98\text{ m/s}$$

7.2.2 $\omega = 2\pi n$

$$\omega = 2\pi(4{,}2)$$

$$\omega = 8{,}4\pi\ rad/s$$

7.2.3 (a) $s_1 = r\theta$

$$s_1 = (7{,}5)\left(160° \times \frac{\pi}{180°}\right)$$

$$s_1 \approx 21 cm$$

(b) $s_2 = r\theta$

$$s_2 = (15)\left(200° \times \frac{\pi}{180°}\right)$$

$$s_2 \approx 52{,}4\ cm$$

(c) $\left(52{,}5\right)^2 = \left(7{,}5\right)^2 + AB^2$

$\text{CD} = \text{AB} \approx 52\ \text{cm}$

Total length = (51,96 + 51,96 + 20,94 + 52,36) cm

$\approx 177{,}3\ \text{cm}$

QUESTION 8

8.1 Hemisphere $Volume = \frac{4}{3}\pi r^3 \div 2$

$Volume = \frac{4}{3}\pi(20)^3 \div 2$

$= 16755{,}16 \text{ m}^3$

Cylinder Volume $= \pi r^2 \times h$

$= \pi(20)^2 \times 8$

$= 10053{,}1 \text{ m}^3$

Volume in one tank = 16755,16 m^3 + 10053,1 m^3

= 26808,26 m^3

Total volume of fuel that the tanker can carry in the 4 tanks = 4(26808,26 m^3)

= 107233,04 m^3

8.2 8.2.1 $a = 120\ m \div 6 = 20\ m$

$Area = a(m_1 + m_2 + m_3 + m_4 + m_5)$

$= 20\left(\frac{12+15}{2} + \frac{15+16}{2} + \frac{16+17}{2} + \frac{17+17}{2} + \frac{17+16}{2} + \frac{16+12}{2}\right)$

$= 20(13{,}5 + 15{,}5 + 16{,}5 + 17 + 16{,}5 + 14)$

$= 1860\ m^2$

8.2.2 Volume = 1 860 $m^2 \times$ 210 m

= 390 600 m^3

8.2.3 Volume for 1 m deep = 120 m × 210 m × 1 m

= 25 200 m^3

1 m^3 = 1 000 litres

25 200 m^3 = 25 200 000 litres

Total: 150 marks