NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2021

TECHNICAL MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

1.1 1.1.1 $2x^2 - x - 6 = 0$

$(2x + 3)(x - 2) = 0$

$x = -\frac{3}{2}$ or $x = 2$

Alternative:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-6)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{1 + 48}}{4}$$

$$\therefore x = 2 \text{ or } x = \frac{-3}{2}$$

1.1.2 $x^2 - 1 = x$

$x^2 - x - 1 = 0$

$$x = 1 \pm \frac{\sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{5}}{2}$$

1.1.3 $4x^2 - 4x + 1 \leq 0$

$(2x - 1)\ (2x - 1) \leq 0$

$(2x - 1)^2 \leq 0$

$$\therefore x = \frac{1}{2}$$

1.2 $3x^2 + 2x + 1 = 0$

$\Delta = 2^2 - 4\,(3)(1)$

$= 4 - 12 = -8$

Roots are non-real

1.3 $3{,}33564095 \times 10^{-5}$

QUESTION 2

2.1 2.1.1 $\sqrt{9x^4 + 16x^4}$

$= \sqrt{25x^4}$

$= 5x^2$

2.1.2 $\left(\dfrac{x^{-\frac{1}{3}}}{\sqrt[3]{x^2}}\right)^{-2}$

$= \left(\dfrac{x^{-\frac{1}{3}}}{x^{\frac{2}{3}}}\right)^{-2}$ OR $(x^{-1})^{-2}$

$= x^2$

$= x^{\frac{2}{3}+\frac{4}{3}}$

$= x^2$

2.2 $\sqrt{5x-1} - 1 = x$

$\sqrt{5x-1} = x + 1$ $x \geq \frac{1}{5};\ x \geq -1$ or check solutions

$5x - 1 = x^2 + 2x + 1$

$0 = x^2 - 3x + 2$

$0 = (x-2)(x-1)$

$x = 2$ or $x = 1$

Both valid

2.3 $\dfrac{2^{2x+3} - 3 \cdot 2^{2x+1}}{2^{x-1}}$

$= \dfrac{2^{2x} \cdot 2^3 - 3 \cdot 2^{2x} \cdot 2^{+1}}{2^x \cdot 2^{-1}}$

$= \dfrac{2^{2x}(8-6)}{2^x \cdot \frac{1}{2}}$

$= 4 \cdot 2^x$

Alternative:

$\dfrac{2^{2x}(2^3 - 3 \cdot 2)}{2^x \cdot 2^{-1}}$

$= \dfrac{2^x(2)}{\cdot 2^{-1}} = 2^x \cdot 4$ **or** 2^{x+2}

2.4 $6 = 3^x$

$\therefore x = \log_3 6$

$= \dfrac{\log 6}{\log 3}$

$\approx 1,6$

Alternative:

$x \log 3 = \log 6$

$\therefore x = \dfrac{\log 6}{\log 3} \approx 1,6$

QUESTION 3

3.1 3.1.1 $w^2 = (a + bi)^2$

$= a^2 + 2abi + b^2i^2$ OR

$= (a^2 - b^2) + 2abi$

3.1.2 $\therefore\ 2ab = -12$ ① and $a^2 - b^2 = 5$ ②

$a = -\dfrac{6}{b}$

Subst. in ② $\left(-\dfrac{6}{b}\right)^2 - b^2 = 5$

$\dfrac{36}{b^2} - b^2 = 5$

$36 - b^4 = 5b^2$

$b^4 + 5b^2 - 36$

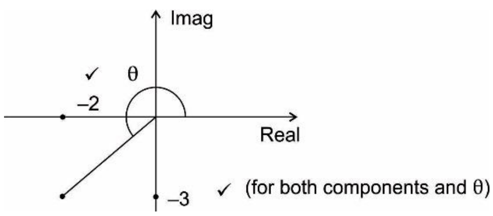
$0 = (b^2 + 9)(b^2 - 4)$

$b^2 = -9$ or $b^2 = 4$

invalid $b = \pm 2$

$\left.\begin{array}{l}\therefore\ b = 2,\ a = -3 \\ \text{or } b = -2,\ a = 3\end{array}\right]$

3.2 $z = -2 - 3i$



$|z|^2 = 4 + 9 = 13$

$|z| = \sqrt{13}$

$\tan\theta = +\dfrac{3}{2}$

$\theta = 236{,}3°$

$z \approx \sqrt{13}\,(\cos 236{,}3° + i \sin 236{,}3°)$

Alternative: $\theta = 4{,}1$ radians

3.3 $$\frac{111_2}{35} = \frac{1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0}{35}$$

$$= \frac{4 + 2 + 1}{35}$$

$$= \frac{7}{35} = \frac{1}{5}$$

QUESTION 4

4.1 Let Value = V

$$\frac{1}{3}\text{V} = \text{V}\,(1 - i)^2$$

$$\sqrt{\frac{1}{3}} = 1 - i$$

$$i = 1 - \sqrt{\frac{1}{3}} \approx 0{,}422649$$

rate ≈ 42,3%

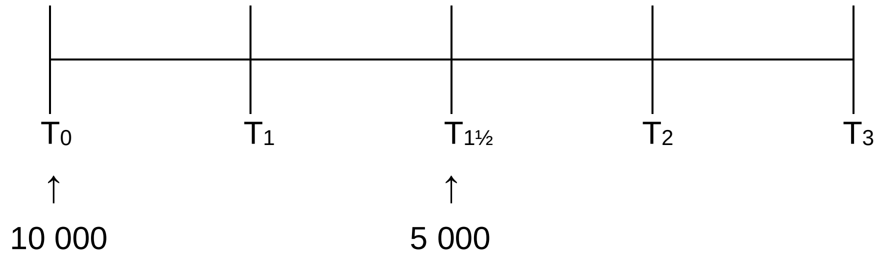
4.2 4.2.1 $1 + i\text{ eff} = \left(1 + \frac{0{,}075}{4}\right)^4$

i eff = 0,07718 …

eff rate = 7,7% p.a.

4.2.2

$i = \frac{0{,}075}{4}$ $i = \frac{0{,}078}{12}$



$$A = 10\,000\left(1 + \frac{0{,}075}{4}\right)^4\left(1 + \frac{0{,}078}{12}\right)^{24} + 5\,000\left(1 + \frac{0{,}078}{12}\right)^{18}$$

≈ R18 201,96

Alternative

(1) **Adding R5 000 after 18 months:**

$$A = \left[10000\left(1 + \frac{0{,}075}{4}\right)^4\left(1 + \frac{0{,}078}{12}\right)^8 + 5000\right] \times \left(1 + \frac{0{,}078}{12}\right)^{1{,}5 \times 12} \approx 18201{,}96$$

(2) **STEP by STEP approach:**

$$A_1 = 10000\left(1+\frac{0{,}075}{4}\right)^4 \approx 10771{,}35868...$$

$$A_2 = (10771{,}35868...)\left(1+\frac{0{,}078}{12}\right)^6 \approx 11198{,}32744...$$

$$A_2 = 11198{,}32744... + 5000 \approx 16198{,}32744...$$

$$A_3 = (16198{,}32744...)\left(1+\frac{0{,}078}{12}\right)^{18} \approx 18201{,}96$$

4.3 $A = P(1 = i)^n$

$$25\,000 \leq 20\,000\left(1+\frac{4}{100}\right)^n$$

$$\frac{5}{4} \leq (1{,}04)^n$$

$$n \geq \log_{1{,}04}\left(\frac{5}{4}\right) \quad \text{OR} \quad n \geq \frac{\log\left(\frac{5}{4}\right)}{\log 1{,}04}$$

$$n \geq 5{,}7 \text{ years}$$

$\therefore$ after 6 years

QUESTION 5

5.1 $y = \frac{a}{x} + b$

$b = -2$

$y = \frac{a}{x} - 2$

Subst $(1; 0)$: $0 = \frac{a}{1} - 2$

$2 = a$

5.2 $y = c\,(x + 4)(x - 2)$

Subst $(-1; 9)$: $9 = c\,(3)(-3)$

$-1 = c$

$y = -1\,(x + 4)(x - 2)$

$y = -x^2 - 2x + 8$

$c = -1$

$d = -2$

$e = 8$

OR

$y = c(x + 1)^2 + 9$

subst. either $(-4 ; 0)$ or $(2 ; 0)$

$\therefore\ 0 = c\,(2 + 1)^2 + 9$

$\therefore\ c = 1$

$\therefore\ y = -1(x + 1)^2 + 9 = -x^2 - 2x - 1 + 9$

$= -x^2 - 2x + 8$

$\therefore\ d = -2$ OR $e = 8$

5.3 $y = f \cdot g^x + h$

Subst $(0; 1)$: $1 = f{\cdot}g^\circ + h$

$1 = f + h$

$h = -3$

$\therefore\ 1 = f - 3$

$4 = f$

$y = 4{\cdot}g^x - 3$

Subst $(1; -1)$: $-1 = 4{\cdot}g^1 - 3$

$2 = 4g$

$\frac{1}{2} = g$

5.4 $x^2 + y^2 = k^2$

Subst $(-3; 4)$: $9 + 16 = k^2$

$k = 5$

QUESTION 6

6.1 Put $y = 0$: $0 = -x^2 - 4x$

$0 = -x(x + 4)$

$x = 0$ or $x_B = -4$ B is (–4; 0)

$x_A = -2$ (by symmetry)

Subst in f: $y = -(-2)^2 - 4(-2)$

$= 4$ A is (–2; 4)

at C, $y = 2^\circ - 8 = -7$ C is (0; –7)

at D, $y = 0$

$0 = 2^x - 8$

$8 = 2^x$

$2^3 = 2^x$

$x = 3$ D is (3; 0)

Alternative:

$f'(x) = 2x - 4 = 0$
$\therefore x = 2$

6.2 BD $= x_D - x_B$

$= 3 - (-4) = 7$ units

AE : at E, $x = -2$

$y = 2^{-2} - 8 = \frac{1}{4} - 8$

$= -\frac{31}{4}$

AE $= y_A - y_E$

$= 4 - \left(-\frac{31}{4}\right)$

$= \frac{47}{4}$ units (or 11,5 units)

6.3 Range of f: $y \in (-\infty; 4]$ OR $y \leq 4$

6.4 $m_{BC} = \dfrac{0+7}{-4-0} = -\dfrac{7}{4}$

i.e. $y = -\dfrac{7}{4}x - 7$

6.5 $x \in (-\infty\,;0)$

6.6 Shift *g* vertically up more than 7 units (past origin) but below A.

QUESTION 7

7.1 $g(x) = \frac{x}{3} - 2$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h}{3} - 2 - \left(\frac{x}{3} - 2\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h-x}{3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{3} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{1}{3}$$

7.2 $y + x = \left(\frac{2}{x} - \sqrt{x}\right)^2 - x$

$$y = \frac{4}{x^2} - \frac{4\sqrt{x}}{x} + x - x$$

$$= 4x^{-2} - 4x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -8x^{-3} + 2x^{-\frac{3}{2}}$$

$$\text{OR} = -\frac{8}{x^3} + \frac{2}{x^{\frac{3}{2}}}$$

7.3 7.3.1 Volume = area of base × height

$300 = x^2y$

$$\frac{300}{x^2} = y$$

7.3.2 Cost in rand = S.A. × Cost/m^2

$= 5(x^2) + 2(4xy)$

$= 5x^2 + 8xy$

$C = 5x^2 + \dfrac{8x \cdot 300}{x^2}$

$= 5x^2 + \dfrac{2\,400}{x}$

7.3.3 $C(x) = 5x^2 + 2\,450x^{-1}$

$C'(x) = 10x - 2\,400x^{-2}$

$= 10x - \dfrac{2\,400}{x^2}$

At min, $10x - \dfrac{2\,400}{x^2} = 0$

$10x^3 = 2\,400 \qquad x \neq 0$

$x^3 = 240$

$x = \sqrt[3]{240}$

Min Cost = $5\left(\sqrt[3]{240}\right)^2 + \dfrac{2\,400}{\left(\sqrt[3]{240}\right)}$

$\approx$ R579,29

QUESTION 8

8.1 $f(x) = -x^3 + 10x^2 - 17x - 28$

$f'(x) = -3x^2 + 20x - 17$

at stat pts, $-3x^2 + 20x - 17 = 0$

$3x^2 - 20x + 17 = 0$

$(3x - 17)(x - 1) = 0$

$x = \frac{17}{3}$ or $x = 1$

$y_E = -1 + 10 - 17 - 28$

$= -36$

E is $(1 ; -36)$

8.2 $x \in \left(1 ;\ \frac{17}{3}\right)$ OR $1 < x < \frac{17}{3}$

8.3 At F; $m_{tan} = f'(5) = -3(25) + 20(5) - 17$

$= -75 + 100 - 17$

$= 8$

Eqn is $y - 12 = 8(x - 5)$ OR $y = 8x - 28$

8.4 $8x - 28 = -x^3 + 10x^2 - 17x - 28$

$x^3 - 10x^2 + 25x = 0$

$x(x^2 - 10x + 25) = 0$

$x(x - 5)^2 = 0$

$x_G = 0$ i.e. y-unit of f

G is $(0 ; -28)$

QUESTION 9

$H = 15 + 3t^2 - \frac{2}{3}t^3$

Rate of change $= \frac{dH}{dt} = +6t - 2t^2$

$\therefore +6t - 2t^2 = -\frac{1}{2}$

$+12t - 4t^2 = -1$

$0 = 4t^2 - 12t + 1$

$t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(1)}}{2(4)}$

$t = 2{,}99$ hours or $0{,}1$ hours

QUESTION 10

10.1 (a) $\int d\theta = \theta + C$

(b) $\int\left(\frac{8}{x} - \frac{5}{x^2} + 6x^3\right)dx = \int\left(\frac{8}{x} - 5x^{-2} + 6x^3\right)dx$

$= 8\ell n(x) - \frac{5x^{-1}}{-1} + \frac{6x^4}{4} = 8\ell n(x) + \frac{5}{x} + \frac{3x^4}{2} + c$

10.2 $\int_0^5 g(x)\,dx = -3$

If $g(x) = g(-x)$

the $\int_{-5}^{0} g(-x)$ = –3 by symmetry

$\therefore \int_{-5}^{5} g(x)$ = –3 + (–3) = –6

10.3 $A = \int_a^b f(x)\,dx = \int_2^4 \left(x^2 - 4\right)dx$

Allocate 1 mark for $a = 2$
Allocate a mark for area application
Allocate 1 mark for integration
Allocate 1 + 1 marks for substitution
Allocate 1 mark for simplification

Solve:

$$\int_2^4 \left(x^2 - 4\right)dx = \left(\frac{1}{3}x^3 - 4x\right)\Bigg|_2^4 = \left(\frac{1}{3}\cdot(4)^3 - 4\cdot 4\right) - \left(\frac{1}{3}\cdot(2)^3 - 4\cdot 2\right)$$

$$= \left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 8\right) = \frac{32}{3}$$ square units

Total: 150 marks