NASIONALE SENIOR SERTIFIKAAT-EKSAMEN
NOVEMBER 2017

WISKUNDE: VRAESTEL I

NASIENRIGLYNE

Tyd: 3 uur | 150 punte

Hierdie nasienriglyne word voorberei vir gebruik deur eksaminatore en hulpeksaminatore. Daar word van alle nasieners vereis om 'n standaardiserings-vergadering by te woon om te verseker dat die nasienriglyne konsekwent vertolk en toegepas word tydens die nasien van kandidate se skrifte.

Die IEB sal geen gesprek aanknoop of korrespondensie voer oor enige nasienriglyne nie. Daar word toegegee dat verskillende menings rondom sake van beklemtoning of detail in sodanige riglyne mag voorkom. Dit is ook voor die hand liggend dat, sonder die voordeel van bywoning van 'n standaardiserings-vergadering, daar verskillende vertolkings mag wees oor die toepassing van die nasienriglyne.

AFDELING A

VRAAG 1

(a) (1)

$$(x-1)^2 = 2(1-x)$$
$$(x-1)^2 = -2(x-1)$$
$$(x-1)^2 + 2(x-1) = 0$$
$$(x-1)(x-1+2) = 0$$
$$(x-1)(x+1) = 0$$
$$x = 1 \qquad x = -1$$

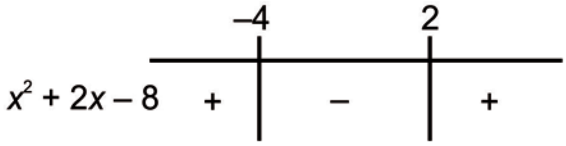
(2)

$$5^{-x}.5^{x-2} = \frac{25^{2x}}{5}$$
$$5^{-x}.5^{x-2} = \frac{5^{4x}}{5^1}$$
$$5^{-x+x-2} = 5^{4x-1}$$
$$-2 = 4x-1$$
$$x = -\frac{1}{4}$$

(b)

$$(x+1)^2 < 9$$
$$\therefore x^2 + 2x + 1 < 9$$
$$\therefore x^2 + 2x - 8 < 0$$
$$\therefore (x+4)(x-2) < 0$$

Kritieke waardes: –4; 2



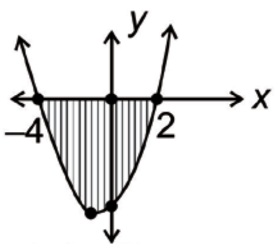
Oplossing: $\{x : -4 < x < 2\}$

Alternatief

$$(x+1)^2 < 9$$
$$x^2 + 2x - 8 < 0$$

Skets: $y = x^2 + 2x - 8$

x-afsnitte: $x = -4$; $x = 2$



Oplossing: $\{x : -4 < x < 2\}$

Alternatief

$$(x+1)^2 < 9$$
$$\therefore -3 < x + 1 < 3$$
$$\therefore -4 < x < 2$$

(c)

$$(x-2)(x+4) = 0$$
$$x^2 + 2x - 8 = 0$$
$$\therefore b = 2 \text{ en } c = -8$$

(d) (1)

$$x-2=\frac{-4}{x-2}-4 \text{ Laat } x-2=y:$$

$$y=-\frac{4}{y}-4 \text{ KGV: } y$$

$$y^2=-4-4y$$

$$\therefore y^2+4y+4=0$$

(2)

$$(y+2)^2=0$$

$$\therefore y=-2$$

Wortels is reëel en gelyk.

Alternatief

$$y^2+4y+4=0$$

$$\therefore \Delta=4^2-4(1)(4)$$

$$\therefore \Delta=0$$

$\therefore$ Wortels is reëel en gelyk.

Alternatief

$$x-2=\frac{-4}{x-2}-4$$

$$\therefore (x-2)^2=-4-4(x-2)$$

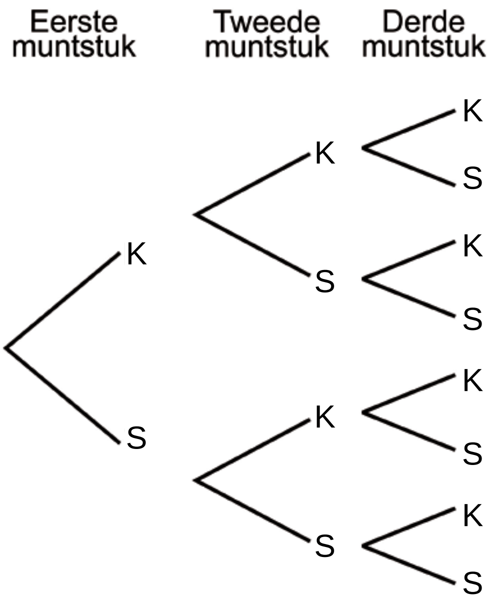
$$\therefore x^2-4x+4=-4-4x+8$$

$$\therefore x^2=0$$

$\therefore$ Wortels is reëel en gelyk.

VRAAG 2

(a) (1)



(2) $E = \{\text{KSS, SKS, SSK}\}$

$\therefore \text{P(2 stert en 1 kop)} = \frac{3}{8}$

(b) (1) $\text{P}(A \cap B) = 0$

(2) (i) Jy kan nie terselfdertyd 'n R2-munt en 'n R5-munt kies nie.

(ii) P(óf 'n R5 óf 'n R2)

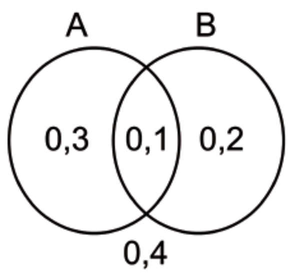
$= \text{P}(\text{A of B})$

$= \text{P(A)} + \text{P(B)}$ onderling uitsluitend

$= 0{,}36 + 0{,}47$

$= 0{,}83$

(c) (1)



(2) P(presies een masjien slaan R5-munte)

$= 0{,}3 + 0{,}2$

$= 0{,}5$

$\therefore 50\%$

VRAAG 3

(a) $480\,163 \div 0{,}502 = \text{R}956\,500$

(b) $\text{R}956\,500 \times 5\% = \text{R}47\,825$

(c) Koste van masjinerie, invoerheffings ingesluit $= \text{R}956\,500 + \text{R}47\,825$

$$= \text{R}1\,004\,325$$

$$\text{A} = \text{P}\,(1+i)^n$$

$$1\,004\,325 = 225\,450\left(1+\frac{9{,}5}{100}\right)^n$$

$$\frac{1\,004\,325}{225\,450} = \left(\frac{219}{200}\right)^n$$

$$\log_{\left(\frac{219}{200}\right)}\left(\frac{1\,004\,325}{225\,450}\right) = n$$

$$n = 16{,}46171594$$

$$\therefore n \approx 16{,}46$$

$\therefore$ ongeveer 17 jaar

(d) (1) Lening verlang: $\text{R}1\,004\,325 - \text{R}225\,450 = \text{R}778\,875$

$$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$778\,875 = x\left[\frac{1-\left(1+\frac{12}{1200}\right)^{(-4\times 12)}}{\frac{12}{1200}}\right]$$

$$x = \text{R}20\,510{,}76607$$

$$\therefore x = \text{R}20\,510{,}77$$

(2) Uitstaande saldo $= A - F$

$$A = 778\,875\left(1+\frac{12}{1200}\right)^{24}$$
$$A = 988\,964{,}5744$$
$$A \approx 988\,964{,}57$$

$$F = 20\,510{,}76607\left[\frac{\left(1+\frac{12}{1200}\right)^{24}-1}{\frac{12}{1200}}\right]$$
$$F = 553\,246{,}4277$$
$$F \approx 553\,246{,}43$$

Uitstaande saldo $= 988\,964{,}5744 - 553\,246{,}4277$
$= \text{R}435\,718{,}1467 \approx \text{R}435\,718{,}15$

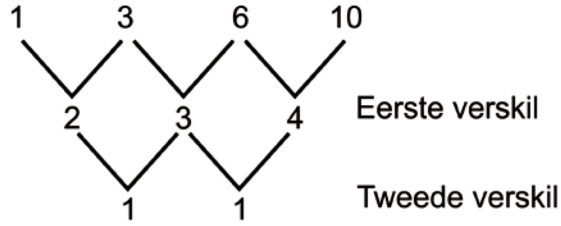
NB: Indien A en F tot die naaste sent afgerond word, oorweeg
Uitstaande saldo $= 988\,964{,}57 - 553\,246{,}43$
$= \text{R}435\,718{,}14$

Alternatief

$$\text{Uitstaande saldo} = 20\,510{,}76607\left[\frac{1-\left(1+\frac{12}{1200}\right)^{-24}}{\frac{12}{1200}}\right]$$
$= \text{R}435\,718{,}1466$
$\approx \text{R}435\,718{,}15$

VRAAG 4

(a) (1)



Konstante tweede verskil

(2) $T_n = an^2 + bn + c$

$T_1 = a + b + c = 1$

$T_2 = 4a + 2b + c = 3$

$T_3 = 9a + 3b + c = 6$

$\therefore 3a + b = 2$ en $5a + b = 3$

Vervang $b = 2 - 3a$ in $5a + b = 3$

$\therefore 5a + (2 - 3a) = 3$

$2a = 1$

$a = \frac{1}{2}$

$b = \frac{1}{2}$ en $c = 0$

$\therefore T_n = \frac{1}{2}n^2 + \frac{1}{2}n$

(b) $T_3 = 52$ cm

$T_7 = 78$ cm

$T_3 = a + 2d = 52$

$T_7 = a + 6d = 78$

$4d = 26 \quad \therefore \quad d = 6\frac{1}{2}$

$\therefore a = 39$ cm

$T_{43} = 39 + 42\left(6\frac{1}{2}\right)$

$T_{43} = 312$ cm

VRAAG 5

(a) (1) $f(x) = x^2 - 6x + 9$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 6(x+h) + 9 - (x^2 6x + 9)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 6x - 6h + 9 - x^2 + 6x - 9}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h(2x + h - 6)}{h}$$

$$f'(x) = 2x - 6$$

(2) $f'(-3) = 2(-3) - 6 = -12$

(b) $y = \pi x^{-1} + 3x^{\frac{1}{3}}$

$$\frac{dy}{dx} = -\pi x^{-2} + x^{-\frac{2}{3}}$$

AFDELING B

VRAAG 6

(a) (1) Definisiegebied $= x \in \mathbb{R} \;;\; x \neq 3$

(2) Waardegebied $= y \in \mathbb{R} \;;\; y \neq -3$

(3) (i) 5 eenhede

(ii) 5 eenhede

(b) (1) $y = a.b^x$ vervang $\left(0;\frac{1}{4}\right)$

$\frac{1}{4} = a.b^0$

$a = \frac{1}{4}$

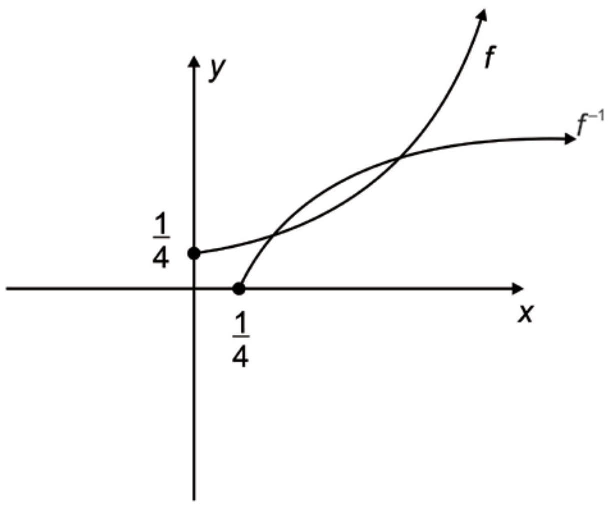
$y = \frac{1}{4}b^x$ vervang $\left(2;\frac{9}{4}\right)$

$\frac{9}{4} = \frac{1}{4}b^2$

$b^2 = 9$

$\therefore b = \pm 3$ maar $b > 0$ $\therefore b = 3$

(2)



(3) Waardegebied = $\left[\frac{1}{4};\infty\right)$

(4) $f(x) = \frac{1}{4}.3^x$

Vir f^{-1}: $x = \frac{1}{4}.3^y \;;\; y \geq 0$

$4x = 3^y$

$y = \log_3(4x)$ vir $x \geq \frac{1}{4}$

(5) Sien grafiek in Vraag 6 (b) (2) hierbo.

VRAAG 7

(a) $f(x) = x^2 + 6x + (3)^2 + 5 - (3)^2$

$f(x) = (x+3)^2 - 4$

$\therefore$ Draaipunt (–3; –4)

(b) (1) $x^2 + 6x + 5 = -x - 5$

$x^2 + 7x + 10 = 0$

$x = -2$ of $x = -5$

$A(-5;0)$ en $B(-2;-3)$

(2) Horisontale skuif: $\therefore -5 < t < -2$

(c) (1) Lengte $\text{MN} = (-x-5) - (x^2 + 6x + 5)$

Lengte $\text{MN} = -x - 5 - x^2 - 6x - 5$

Lengte $\text{MN} = x^2 - 7x - 10$

Vir maksimum lengte: Laat $D_x = 0$

$-2x - 7 = 0$

$x = -\frac{7}{2}$

$\therefore$ Maks. lengte $\text{MN} = -\left(-\frac{7}{2}\right)^2 - 7\left(-\frac{7}{2}\right) - 10$

$\therefore$ Maks. lengte $\text{MN} = \frac{9}{4}$ eenhede d.w.s. $2{,}25$ eenhede

(2) Vertikale skuif: $\therefore k > \frac{9}{4}$

VRAAG 8

(a) (1) $\frac{3}{2}\ ;-\frac{9}{2}\ ;\frac{27}{2}\ ;...$

$\therefore r=-3$ en reeks is meetkundig

Reeks is egter nie konvergent nie, aangesien $r<-1$.

$\therefore x\neq\frac{3}{2}$

(2) $\frac{x-3}{x+3}=\frac{12-x}{x-3}$

$(x-3)^2=(12-x)(x+3)$

$x^2-6x+9=12x+36-x^2-3x$

$2x^2-15x-27=0$

$x=9$ of $x\neq-\frac{3}{2}$

(b) $S_4=7\frac{1}{2}\ ;S_5=15\frac{1}{2}$ en $S_6=31\frac{1}{2}$

$T_5=S_5-S_4$

$T_5=8$

$T_6=S_6-S_5$

$T_6=16$

$T_5=ar^4=8$

$T_6=ar^5=16$

$\frac{T_6}{T_5}=r=2$

$\therefore a=\frac{1}{2}$

$S_n=\frac{\frac{1}{2}(2^n-1)}{2-1}$

$=2^{n-1}-\frac{1}{2}$

VRAAG 9

(a) $f(x) = -x^3 + bx^2 + cx - 3$

$f(1) = -(1)^3 + b(1)^2 + c(1) - 3 = 4$

$b + c = 8$

$f'(x) = -3x^2 + 2bx + c$

$f''(x) = -6x + 2b$

$f''\left(\frac{1}{2}\right) = -6\left(\frac{1}{2}\right) + 2b = 1$

$b = 2$

$\therefore c = 6$

(b) Vir konkaaf boontoe: $f''(x) > 0$

$-6x + 4 > 0$

$x < \frac{2}{3}$

VRAAG 10

$$\frac{340}{x} - \frac{340}{x+2} = 3 \quad \text{KGV: } x(x+2)$$

$$340(x+2) - 340x = 3x(x+2)$$

$$3x^2 + 6x - 680 = 0$$

$$x = 14,09 \quad \text{of} \quad x \neq -16,09$$

Daarom is Tyd = $\frac{340}{14,09} \approx 24,13$ sekondes

Alternatief

Laat oorspronklike tyd geneem voorgestel word deur y.

$$\therefore xy = 340 \text{ ... verg. 1}$$

$$(x+2)(y-3) = 340 \text{ ... verg. 2}$$

Uit verg. 1 $\quad y = \frac{340}{x}$

$$\therefore (x+2)\left(\frac{340}{x} - 3\right) = 340$$

$$\therefore 3x^2 + 6x - 680 = 0$$

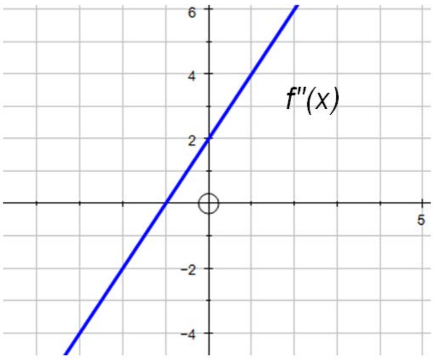
$$\therefore x = 14,09 \text{ of } x \neq -16,09$$

Daarom is Tyd = $\frac{340}{14,09} \approx 24,13$ sekondes

VRAAG 11

(a) (1) Wanneer $x = -2$ en $x = 0$

(2)



(b) $y = \frac{1}{5}x^3 + \frac{3}{4}x + 3$

$\frac{dy}{dx} = \frac{3}{15}x^2 + \frac{3}{4}$ vervang $x = 0$

$\frac{dy}{dx} = \frac{3}{4}$

Vergelyking van raaklyn: $y = \frac{3}{4}x + c$ waar $c = 3$

Vergelyking van raaklyn: $y = \frac{3}{4}x + 3$

Vir snypunt tussen raaklyn

en lyn BC, vervang $x = 2$ in $y = \frac{3}{4}x + 3$

$\therefore y = 4\frac{1}{2} \therefore \text{Pt}\left(2;4\frac{1}{2}\right)$

Oppervlakte van Busi se gebied $= \frac{1}{2}\left(5 + 3\frac{1}{2}\right) \times 2$

$= 8\frac{1}{2}$ eenhede^2

Oppervlakte van Khanya se gebied $= \frac{1}{2}\left(3 + 4\frac{1}{2}\right) \times 2$

$= 7\frac{1}{2}$ eenhede^2

Derhalwe is Busi se gebied die grootste.

Totaal: 150 punte