NASIONALE SENIORSERTIFIKAAT-EKSAMEN
NOVEMBER 2016

WISKUNDE: VRAESTEL I

NASIENRIGLYNE

Tyd: 3 uur 150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en sub-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar, sonder die voordeel van die bywoning van 'n standaardiseringsvergadering, verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

AFDELING A

VRAAG 1

(a) (1) $\frac{4x}{2} - \frac{2x+1}{3} = 5$

$\frac{12x - 4x - 2}{6} = \frac{30}{6}$ **OF** $12x - 2(2x + 1) = 30$

$8x = 32$

$x = 4$

(2) $(x-5)(x-6) \leq 56$

$x^2 - 11x + 30 \leq 56$

$x^2 - 11x - 26 \leq 0$

$(x-13)(x+2) \leq 0$

Kritieke waardes: 13; –2

$-2 \leq x \leq 13$

(b) Draaipunt (–2; –8) Vorm

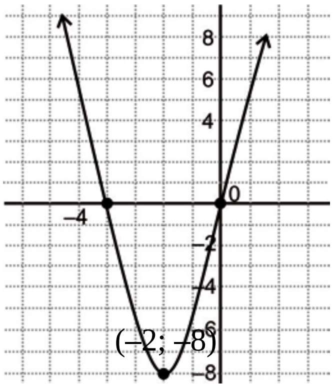
Y-afsnit: (0; 0)

X-afsnitte: Laat $y = 0$.

$(x + 2) = \pm 2$ OF $x(x + 4) = 0$

$\therefore x = 0$ **OF** $x = -4$

$\therefore x$-afsnitte: (0; 0) en (–4; 0)



(c) (1) $x = -1$

en

$y = 2$

(2) $\frac{4}{x+1} + 2 = x$

$\therefore 4 + 2(x + 1) = x(x + 1)$

$\therefore 4 + 2x + 2 = x^2 + x$

$\therefore x^2 - x - 6 = 0$

$\therefore (x - 3)(x + 2) = 0$

$\therefore$ (3; 3) (2; 2)

NB: (Vir $x = 3$ **OF** $x = 2$ ken 3 uit 4 toe)

(d) $c = -1$ of $c = -\frac{1}{4}$ (ander antwoorde moontlik)

(e) $3 - k < 0 \therefore k > 3$

VRAAG 2

(a) (1) $LK = 3\left(\frac{1}{3}\right) = 1$

$RK = \sqrt{6\left(\frac{1}{3}\right) - 1} = -1$

$LK \neq RK \therefore x = \frac{1}{3}$ is verkeerd

(2) $3x = -\sqrt{6x-1}$

$(3x)^2 = 6x - 1$

$9x^2 - 6x + 1 = 0$

$x = \frac{1}{3}$

uit (1), geen oplossing nie

Alternatief: Laat x 'n oplossing wees.

Dan $3x < 0$ dus $x < 0$

Maar $x \geq \frac{1}{6}$

$\therefore$ Geen oplossing nie

(b) $7^{x+a}(1+3) = 28\left(7^{a^2}\right)$

$7^{x+a} = \frac{28\left(7^{a^2}\right)}{4}$

$7^{x+a} = 7\left(7^{a^2}\right)$

$7^{x+a} = 7^{1+a^2}$

$x = a^2 - a + 1$

VRAAG 3

(a) $4\,800 - \left(4\,800 \times \frac{13,5}{100}\right)$

$= R4\,152$

(b) $415\,200 = x\left[\frac{1-\left(1+\frac{7}{1\,200}\right)^{(-5\times 12)}}{\frac{7}{1\,200}}\right]$ Gebruik korrekte formule

$x \approx R8\,221,46$

VRAAG 4

(a) Bedrag betaal vir al die 110 skootrekenaars: 6 000 × 110 = 660 000

Depresiasie oor 5 jaar: $A = 660\,000\left(1 - \frac{15}{100}\right)^5$

$\approx 292\,845{,}51$

Inflasie: $A = P(1+i)^n$

$A = 660\,000\left(1 + \frac{6}{100}\right)^5$

$A = 883\,228{,}881$

Bedrag oor 5 jaar benodig minus "terugkoop" = 883 228,88 – 292 845,51

= R590 383,37

(b) Delgingsfonds: $F = x\left[\frac{(1+i)^n - 1}{i}\right]$

$590\,383{,}37 = x\left[\frac{\left(1+\frac{12}{1200}\right)^{(5\times 12)} - 1}{\frac{12}{1200}}\right]$ Gebruik korrekte formule

$x \approx \text{R}7\,228{,}92$

VRAAG 5

(a) $T_1 = 5(1) + 2 = 7$

$T_2 = 5(2) + 2 = 12$

$T_3 = 5(3) + 2 = 17$

$\therefore$ Aangesien $T_1 + T_2 + T_3 = 36$

Dan $y = 3$

OF **Alternatief:**

$7 + 12 + 17 + ... + (5y + 2)$

$\therefore$ ry is rekenkundig

met $a = 7$ en $d = 5$

$\therefore \frac{y}{2}[7 + 5y + 2] = 36$

$\therefore 9y + 5y^2 = 72$

$\therefore 5y^2 + 9y - 72 = 0$

$\therefore (5y + 24)(y - 3) = 0$

$\therefore y = 3$

(b) (1) $3p - (2p + 14) = (p + 7) - 3p$

$3p - 2p - 14 = p + 7 - 3p$

$3p = 21$

$p = 7$

(2) $a = 28$ en $d = -7$

$S_{38} = \frac{38}{2}\left[2(28) + (38-1)(-7)\right]$

$S_{38} = -3\,857$

(c) $T_n = an^2 + bn + c$

$a + b + c = 7$ … eq ①

$4a + 2b + c = 13$ … eq ②

$9a + 3b + c = 21$ … eq ③

② – ①: $3a + b = 6$

③ – ②: $5a + b = 8$

Sub. $b = 6 - 3a$

Into: $5a + b = 8$

$5a + 6 - 3a = 8 \quad \therefore\ 2a = 2$

$a = 1,\ b = 3,\ c = 3$

$\therefore\ T_n = n^2 + 3n + 3$

Alternatief 1

$$T_n = \frac{(n+1)^3 - 1}{n}$$

$$= \frac{n^3 + 3n^2 + 3n + 1 - 1}{n}$$

$$= n^2 + 3n + 3$$

(d) $r = \dfrac{2}{3}$

Die ry van somme is:

$9,\ 15;\ 19;\ 21\frac{2}{3};\ 23\frac{4}{9};\ \frac{665}{27};\ \frac{2\,059}{81}$

$T_6 \approx 24{,}6$ en $T_7 = 25{,}4$

$\therefore\ n = 7$ is die kleinste.

OF

$$Sn = 27\left(1 - \frac{2}{3}\right)^n$$

$Sn > 25$ lei tot $\left(\frac{2}{3}\right)^n < \frac{2}{27}$

Probeer $n = 6$, dit werk nie.

Maar $n = 7$ werk.

Alternatief

$a = 9$ en $r = \dfrac{2}{3}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$25 = \frac{9\left[\left(\frac{2}{3}\right)^n - 1\right]}{\frac{2}{3} - 1}$$

$$\left(\frac{2}{3}\right)^n = \frac{2}{27}$$

$$n = \log_{\frac{2}{3}} \frac{2}{27}$$

$n = 6{,}41\ ...$

$\therefore$ Kleinste waarde: $n = 7$

(e) $V_1 = 729 \text{ cm}^3$

$V_{n+1} = \frac{1}{3} A_{n+1} \cdot h_{n+1}$

$= \frac{1}{3}\left(\frac{1}{3}\right) An \left(\frac{1}{3} h_n\right)$

$= \frac{1}{9} V_n$

$\therefore$ Ry is meetkundig

met $r = \frac{1}{9}$

$\therefore S\infty = \frac{729}{1 - \frac{1}{9}} = 820{,}1 \text{ cm}^3$

Alternatief

Volume van piramide ① = $\frac{1}{3} \times (9 \times 9) \times 27 = 729 \text{ cm}^3$

Volume van piramide ② = $\frac{1}{3} \times \left(\frac{81}{3}\right) \times \frac{27}{3} = 81 \text{ cm}^3$

Volume van piramide ③ = $\frac{1}{3} \times \left(\frac{27}{3}\right) \times \frac{9}{3} = 9 \text{ cm}^3$

Die ry is meetkundig.

$a = 729$; gemene verhouding is $\frac{1}{9}$

$S\infty = \frac{a}{1 - r}$

$= 820\frac{1}{8} \text{ cm}^3$

VRAAG 6

(a) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Berekening:

$$f(x) = 3x^2 + 2x$$

$$f(x+h) = 3(x+h)^2 + 2(x+h)$$

$$f(x+h) = 3x^2 + 6xh + 3h^2 + 2x + 2h$$

$$f'(x) = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h - (3x^2 + 2x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$f'(x) = \lim_{h \to 0} (6x + 3h + 2)$$

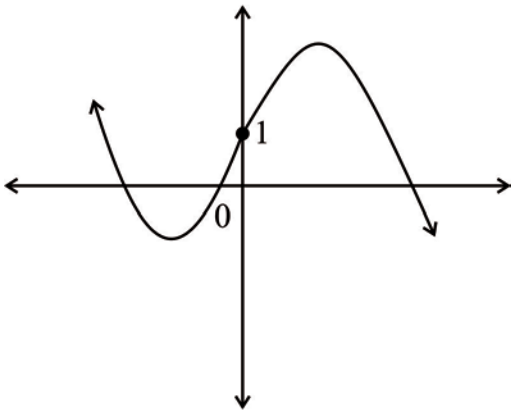
$$f'(x) = 6x + 2$$

(b) $$y = -x^{-1} + x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = +x^{-2} + \frac{1}{2} x^{-\frac{1}{2}}$$

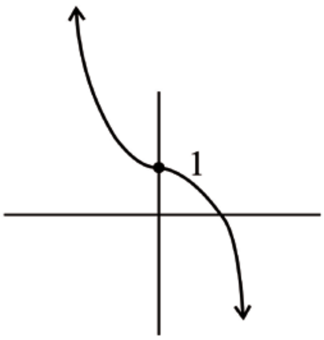
AFDELING B

VRAAG 7



Vorm
y-afsnit en buigpunt (0; 1)
Konkaaf ondertoe vir $x > 0$

OF



Vorm
y-afsnit en buigpunt (0; 1)
Konkaaf ondertoe vir $x > 0$

VRAAG 8

(a) Simmetrie-as: $x = \dfrac{-3+1}{2} = -1$

$\therefore f'(x) > 0$ en $g(x) < 0$
OF $f'(x) < 0$ en $g(x) > 0$
$\therefore x < -1$ **OF** $x > 0$ (4)

(b) $g(x) = d^x + q$ vervang (0; 0)

$0 = d^0 + q$

$q = -1$

Vervang (1; 2)

$2 = d^1 - 1$

$d = 3$

$\therefore g(x) = 3^x - 1$

(c) Inverse van g:

$x = 3^y - 1$

$3^y = x + 1$

$y = \log_3(x+1)$

(d) Definisiegebied: $x > -1$

Alternatief

Definisiegebied van g^{-1} = Waardegebied van g
= $(-1;\ \infty)$

(e) $f(x) = a(x + 3)(x - 1)$

$6 = a(3)(-1)$

$A = -2$

$\therefore f(x) = -2(x + 3)(x - 1)$

$= -2x^2 - 4x + 6$

$\therefore a = -2,\ b = -4,\ c = 6$

Alternatief: Gegee y-afsnit (0;6)

$y = ax^2 + bx + 6$ sub (-3;0)

$0 = a(-3)^2 + b(-3) + 6$

$b = \frac{9a+6}{3}$ eq. 1

sub (1;0) $0 = a(1)^2 + b(1) + 6$

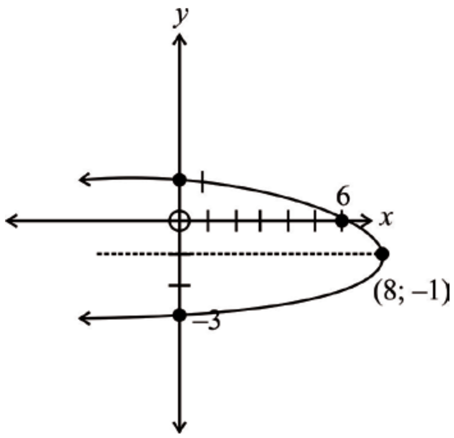
$a + b + 6 = 0$ eq. 2

Sub. Eq. 1 in Eq. 2

$-24 = 12a \quad \therefore \quad a = -2$

$b = -4$

(f)



Draaipunt van f: $f(-1) = -2(-1 + 3)(-1 - 1)$
$= 8$
$\therefore$ (8; –1) is draaipunt van g

Vorm

y-afsnit: –3 en 1

x-afsnit: 6

Draaipunt (8; –1)

(g) $k > -6$ **OF** $k \in (-6\ ;\ \infty)$

VRAAG 9

(a) $f(1) = a(1)^3 + b(1)^2 \therefore a + b$

$f(2) = a(2)^3 + b(2)^2 \therefore f(2) = 8a + 4b$

$\therefore 5{,}5 = \dfrac{8a + 4b - (a+b)}{2-1}$

$7a + 3b = 5{,}5$ …… (1)

$f'(x) = 3ax^2 + 2bx$

$-18 = 3a(6)^2 + 2b(6)$

$-18 = 108a + 12b$ …… (2)

4(1)–(2): $\begin{cases} 28a + 12b = 22 \\ 108a + 12b = -18 \end{cases}$

$\therefore -80a = 40$

$\therefore a = -\dfrac{1}{2}$

$\therefore b = 3$

Let wel: Geen punte vir slegs antwoord nie.

(b) $f(x)$ is stygend wanneer $f'(x) \geq 0$

$-\dfrac{3}{2}x^2 + 6x \geq 0$

$-3x^2 + 12x \geq 0$

$-3x(x-4) \geq 0$

$0 \leq x \leq 4$

Alternatief

$f'(x) = -\dfrac{3}{2}x^2 + 6x$

$= \dfrac{-3x}{2}(x-4)$

$\therefore x_c = 4$

f is stygend op $0 \leq x \leq x_c = 4$

(c) f is konkaaf ondertoe wanneer

$f''(x) < 0$

$-6x + 12 < 0$

$x > 2$

Alternatief

Buigpunt is: $x = \dfrac{0+4}{2} = 2$

Uit grafiek is f konkaaf ondertoe wanneer $x > 2$

VRAAG 10

$h + r = 9$
$\therefore h = 9 - r$

$V = \pi r^2 h$
$V = \pi r^2 (9 - r)$
$V = 9\pi r^2 - \pi r^3$

$V = 9\pi r^2 - \pi r^3$
$V' = 18\pi r - 3\pi r^2$
$0 = 18\pi r - 3\pi r^2$
$3\pi r(6 - r) = 0$
$r \neq 0 \quad \therefore \quad r = 6$ units

VRAAG 11

(a) (1) $\dfrac{46}{80} \times \dfrac{45}{79} = 0,3$

(2) $\left(\dfrac{9}{80} \times \dfrac{25}{79}\right) + \left(\dfrac{25}{80} \times \dfrac{9}{79}\right)$
$= \dfrac{45}{1264} + \dfrac{45}{1264}$
$= \dfrac{45}{632} \approx 0,07$

(b) $\dfrac{8!}{2!2!}$
$= 10\,080$

(c) $P(\text{Khanya sal wen}) = P(RB) + P(RRRB) + P(RRRRRB) + ...$

$P(\text{Khanya sal wen}) = \left(\dfrac{6}{7} \times \dfrac{1}{7}\right) + \left[\left(\dfrac{6}{7}\right)^3 \times \dfrac{1}{7}\right] + \left[\left(\dfrac{6}{7}\right)^5 \times \dfrac{1}{7}\right] + \ldots$

Dit is 'n oneindige meetkundige reeks aangesien $-1 < r < 1$.

$a = \dfrac{6}{7^2}$ en $r = \left(\dfrac{6}{7}\right)^2$

Vir $\frac{1}{6}$ en $\frac{1}{7}$
Boomdiagram

$P(\text{Khanya sal wen}) = \dfrac{a}{1 - r}$

$P(\text{Khanya sal wen}) = \dfrac{\dfrac{6}{7^2}}{1 - \left(\dfrac{6}{7}\right)^2} = \dfrac{6}{13} \approx 0,46$

VRAAG 12

$$\left[\frac{1}{8}(4\pi x^2)+(x^2-\frac{1}{4}\pi x^2)\times 3\right]\times 8+30x^2 = 28$$

$$\therefore 24x^2-2\pi x^2+30x^2 = 28$$

$$x^2(54-2\pi) = 28$$

$$\therefore x^2 = \frac{28}{54-2\pi}$$

$$\therefore x = 0,766$$

$$\therefore x \approx 0,8$$

Punte as volg toegeken:

$54x^2$
$30x^2$
$4\pi x^2$
$6(24x^2-6\pi x^2)$
Som van die drie dele = 28
$\therefore x \quad \approx 0,8$

Totaal: 150 punte