NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2021

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

NOTE:

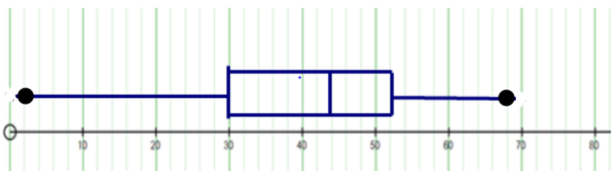
- If a candidate answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

SECTION A

QUESTION 1

(a)	$A = -160,645$ $B = 21,505$ $y = -160,645 + 21,505x$	$A = -160,645$ $B = 21,505$ rounding
(b)	$y = -160,645 + 21,505(90)$ $y = 1774,81$ **Alternate with calculator:** R 1774,79	R1 774,81 Alt: R 1774,79
(c)	Extrapolation has its risks, i.e. when working outside the boundaries of the given data.	Extrapolation
(d)	$r = 0,912$	$r = 0,912$
(e)	Very strong positive correlation	Very strong positive correlation

QUESTION 2

(a)	 Correct box and whisker plot accordingly	Shape: Box & Whisker Min: 2 Max: 68 Q1: 30 Q2: 44 Q3: 52 Max. 2 if box & whisker has errors
(b)	Skewed left / negatively skewed	negatively skewed
(c)	Since Range A > Range B and $IQR_A > IQR_B$, the heights of the plants grown in Environment A were more spread out.	as described

QUESTION 3

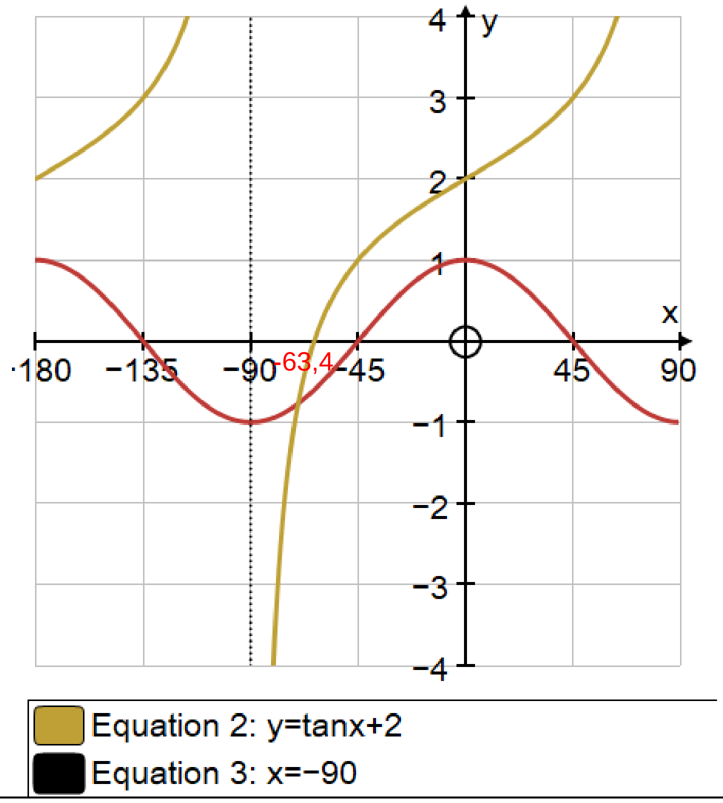
(a)	Length AB $=\sqrt{\left(x^2-x_1\right)^2+\left(y_2-y_1\right)^2}$ Length AB $=\sqrt{(11-6)^2+(12-16)^2}$ Length AB $=\sqrt{25+16}$ Length AB $=\sqrt{41}$	$=\sqrt{(11-6)^2+(12-16)^2}$ Sub in dist. formula $=\sqrt{41}$
(b)	$m_{AB}=\dfrac{16-12}{6-11}$ $m_{AB}=-\dfrac{4}{5}$ $m_{DE}=\dfrac{-11+3}{6+4}=-\dfrac{8}{10}$ $m_{DE}=-\dfrac{4}{5}$ Gradients are equal $\therefore$ AB//DE	Gradients $m_{AB}=-\dfrac{4}{5}$ $m_{DE}=-\dfrac{4}{5}$
(c)	Eq. of line DB: $y = mx + c$ sub. (m_{DB} = 1) $y=x+c$ sub. (–4;–3) or (11;12) $-3=-4+c$ $c=1$ $\therefore y=x+1$ For point of int. sub. $x=6$ $\therefore y=7$ $\therefore k=7$	sub. (m_{DB} = 1) $c=1$ $x=6$ $\therefore y=7$
(d)	$m_{AB}=-\dfrac{4}{5}$ $\tan\theta=m$ $\theta\approx 38{,}7^\circ$ $AE\perp$ x-axis $\therefore \alpha=90^\circ$ $B\hat{A}C=180^\circ-(90^\circ+38{,}7^\circ)$ (int. $\angle$ of Δ) $B\hat{A}C=51{,}3^\circ$	$\theta\approx 38{,}7^\circ$ $AE\perp$ x-axis $\therefore \alpha=90^\circ$ $B\hat{A}C=51{,}3^\circ$

(e)	$\frac{\text{Area }\Delta ABC}{\text{Area }\Delta EDC}=\frac{\frac{1}{2}(AB)(BC)\sin\hat{B}}{\frac{1}{2}(CD)(DE)\sin\hat{D}}$
	$\Delta ABC///\Delta EDC$ (equiangular)
	$\therefore \frac{AB}{ED}=\frac{BC}{DC}=\frac{AC}{EC}$
	Since $\hat{D}=\hat{B}$ (alt $\angle$s; // lines)
	and $\frac{AB}{DE}=\frac{BC}{DC}$ (/// Δs, sides in Prop)
	$\therefore \frac{\text{Area }\Delta ABC}{\text{Area }\Delta EDC}=\frac{(AB)^2}{(DE)^2}$
	$\therefore \frac{\text{Area }\Delta ABC}{\text{Area }\Delta EDC}=\frac{(\sqrt{41})^2}{(2\sqrt{41})^2}$
	$\therefore \frac{\text{Area }\Delta ABC}{\text{Area }\Delta EDC}=\frac{1}{4}$
	
	Alternate 1:
	$\frac{\text{Area }\Delta ABC}{\text{Area }\Delta EDC}=\frac{\frac{1}{2}(AC)(AB)\sin\hat{A}}{\frac{1}{2}(CE)(ED)\sin\hat{E}}$
	$\frac{\text{Area }\Delta ABC}{\text{Area }\Delta EDC}=\frac{9\times\sqrt{41}}{18\times 2\sqrt{41}}=\frac{1}{4}$
	
	Alternate 2:
	$\frac{\text{Area }\Delta ABC}{\text{Area }\Delta EDC}=\frac{\frac{1}{2}(AC)(h_B)}{\frac{1}{2}(CE)(h_D)}$
	$\frac{\text{Area }\Delta ABC}{\text{Area }\Delta EDC}=\frac{9\times 5}{18\times 10}=\frac{1}{4}$ … $h_B=12-7$
		$=\frac{\frac{1}{2}(AB)(BC)\sin\hat{B}}{\frac{1}{2}(CD)(DE)\sin\hat{D}}$
		$\hat{D}=\hat{B}$ (alt $\angle$s; // lines)
		$\frac{AB}{DE}=\frac{BC}{DC}$ (/// Δs, sides in Prop)
		$\frac{\text{Area }\Delta ABC}{\text{Area }\Delta EDC}=\frac{(AB)^2}{(DE)^2}$
		$\frac{\text{Area }\Delta ABC}{\text{Area }\Delta EDC}=\frac{1}{4}$
		
		$\frac{\frac{1}{2}(AC)(AB)\sin\hat{A}}{\frac{1}{2}(CE)(ED)\sin\hat{E}}$
		Cancelling
		$\frac{9\times\sqrt{41}}{18\times 2\sqrt{41}}$
		$=\frac{1}{4}$
		
		$\frac{\frac{1}{2}(AC)(h_B)}{\frac{1}{2}(CE)(h_D)}$
		Perp heights 5 and 10
		Values 9 and 18
		$\frac{9\times 5}{18\times 10}$
		$=\frac{1}{4}$

QUESTION 4

(a)(1)	Length AB $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ Length AB $=\sqrt{(2-1)^2+(8+1)^2}$ Length AB $=\sqrt{1+81}$ Length AB $=\sqrt{82}$ **Alternate:** $\lvert AB\rvert=\sqrt{82}$	$=\sqrt{(2-1)^2+(8+1)^2}$ Sub in dist. formula $=\sqrt{82}$ $\lvert AB\rvert=\sqrt{82}$
(a)(2)	AB is a diameter: For centre: MidPt AB$\left(\frac{2+1}{2};\frac{8-1}{2}\right)$ MidPt AB$\left(\frac{3}{2};\frac{7}{2}\right)$ $r=\frac{\sqrt{82}}{2}$ $\left(x-\frac{3}{2}\right)^2+\left(y-\frac{7}{2}\right)^2=\frac{41}{2}$	MidPt AB$\left(\frac{3}{2};\frac{7}{2}\right)$ $r=\frac{\sqrt{82}}{2}$ $\left(x-\frac{3}{2}\right)^2+\left(y-\frac{7}{2}\right)^2=\frac{41}{2}$
(a)(3)	$m_{diam}=\frac{8+1}{2-1}\quad\therefore\quad m_{diam}=9$ $\therefore\quad m_{\tan}=-\frac{1}{9}$ $y=-\frac{1}{9}x+c$ sub. $(2;8)$ $c=8\frac{2}{9}$ $y=-\frac{1}{9}x+8\frac{2}{9}$ $9y=-x+74$	$m_{diam}=\frac{8+1}{2-1}\quad\therefore\quad m_{diam}=9$ $\therefore\quad m_{\tan}=-\frac{1}{9}$ $c=8\frac{2}{9}$ $9y=-x+74$
(b)	Construct AO $\therefore$ AO=10 units …. Radius AO$\perp$AM …. Tan $\perp$ Rad $(\text{AM})^2=(13)^2-(10)^2$ …. Pythag AM $=\sqrt{69}$	AO=10 units AO$\perp$AM ..Tan $\perp$ Rad $(\text{AM})^2=(13)^2-(10)^2$ AM $=\sqrt{69}$

QUESTION 5

<table>
<tr>
<td>(a)</td>
<td>


</td>
<td>f(x):
shape & (endpoints)
turning points
x-intercepts

g(x):
shape & (endpoints)
asymptote
x-intercept $(63,4^\circ)$

Do not penalise twice for incorrect endpoints</td>
</tr>
<tr>
<td>(b)</td>
<td>$\cos 2x \leq \tan x + 2$
$x \in [-180^\circ;-90^\circ) \cup [-70,1^\circ;90^\circ)$

Allow range of $[-65^\circ;-75^\circ]$ for point of intersection – reading from graph</td>
<td>$[-180^\circ;-90^\circ)$
$[-70,1^\circ;90^\circ)$</td>
</tr>
</table>

QUESTION 6

(a)	Construction: B through centre O Proof: $\hat{O}_1 = \hat{A} + \hat{B}_1$ (ext. $\angle$ of Δ) $\hat{A} = \hat{B}_1$ (Isos Δ / Radii) Similarly, in the other triangle: $\hat{O}_1 = 2 \times \hat{B}_1$ $\hat{O}_2 = 2 \times \hat{B}_2$ $\therefore A\hat{O}C = 2 \times A\hat{B}C$	B through centre O $\hat{O}_1 = \hat{A} + \hat{B}_1$ (ext. $\angle$ of Δ) $\hat{A} = \hat{B}_1$ (Isos Δ / Radii) $\hat{O}_1 = 2 \times \hat{B}_1$ $\hat{O}_2 = 2 \times \hat{B}_2$ $\therefore A\hat{O}C = 2 \times A\hat{B}C$
(b)(1)	$\hat{C}_1 = \hat{A}_1$ (Tan. from pt / isosceles Δ) $2\hat{A}_1 + \hat{T} = 180°$ $\hat{A}_1 = 59°$ (Int. $\angle$s of Δ)	$\hat{A}_1 = 59°$ (Tan. from pt / isosceles Δ) (Int. $\angle$s of Δ)
(b)(2)	$\hat{A}_1 + \hat{A}_2 = 90°$ (radius $\perp$ tangent) $\hat{A}_2 = 90° - 59°$ $\hat{A}_2 = 31°$ $\hat{A}_2 = \hat{C}_2$ (Isos. Δ; CO=AO radii) $\therefore \hat{O}_1 = 118°$ (int. $\angle$s of Δ) **ALTERNATE:** $\hat{A}_1 = \hat{B}$ (Tan-Chord Th) $\hat{A}_1 = 59°$ (From (b)(1)) $\therefore \hat{O}_1 = 118°$ ($\angle$ at centre $= 2$ X $\angle$ at cir)	$\hat{A}_1 + \hat{A}_2 = 90°$ (radius $\perp$ tangent) $\hat{A}_2 = 90° - 59°$ $\hat{A}_2 = 31°$ $\hat{A}_2 = \hat{C}_2$ $\therefore \hat{O}_1 = 118°$ (int. $\angle$s of Δ) $\hat{A}_1 = \hat{B}$ (Tan-Chord Th) $\hat{A}_1 = 59°$ (From (b)(1)) $\therefore \hat{O}_1 = 118°$ ($\angle$ at centre $= 2$ X $\angle$ at cir)

QUESTION 7

(a)	DO=3 units AD:DO = 4 : 3 $\frac{AD}{DO}=\frac{AE}{EC}=\frac{AF}{FB}$ (Prop Th – DE//OC & EF//CB) $\therefore$ AF : FB = 4 : 3	AD:DO = 4 : 3 $\frac{AD}{DO}=\frac{AE}{EC}=\frac{AF}{FB}$ with reason $\therefore$ AF : FB = 4 : 3
(b)	$\Delta AHF /// \Delta AGB$ (Equiangular) $\therefore \frac{AH}{AG}=\frac{HF}{GB}=\frac{AF}{AB}$ (similar triangles, sides in prop) $AB=7x$ $\therefore \frac{HF}{GB}=\frac{4x}{7x}$ $\therefore GB:HF=7:4$	$\Delta AHF /// \Delta AGB$ with reason $\therefore \frac{AH}{AG}=\frac{HF}{GB}=\frac{AF}{AB}$ with reason $\therefore GB:HF=7:4$
(c)	AE:EC = 4 : 3 (Prop Th) EG = GC = $1\frac{1}{2}k$ $\therefore$ AE : EG $=4:\frac{3}{2}$ or $8:3$	AE:EC = 4 : 3 (Prop Th) EG = GC = $1\frac{1}{2}k$ $\therefore$ AE : EG $=4:\frac{3}{2}$ or $8:3$

SECTION B

QUESTION 8

(a)	$\sin 3x = -\frac{3}{4}$ $3x = -48,6° + k360°$; $k \in Z$ $x = -16,2° + k120°$; $k \in Z$ or $3x = 180 - (-48,6°) + k360°$; $k \in Z$ $x = 76,2° + k120°$; $k \in Z$ $x = \{-16,2°; -43,8°\}$	$3x = -48,6° + k360°; k \in Z$ $x = -16,2° + k120°; k \in Z$ $x = 76,2° + k120°$; $k \in Z$ Quadrants $x = \{-46,2°; -73,8°\}$
(b)	$\tan x = \sin 2x$ $\frac{\sin x}{\cos x} = 2\sin x \cos x$ $\sin x = 2\sin x \cos^2 x$ $2\sin x \cos^2 x - \sin x = 0$ $(2\cos^2 x - 1) = 0$ $\cos 2x = 0$ $2x = \pm 90° + k360°$; $k \in Z$ $\therefore x = \pm 45° + k180°$; $k \in Z$ **Alternate:** $\tan x = \sin 2x$ $\frac{\sin x}{\cos x} = 2\sin x \cos x$ $\sin x = 2\sin x \cos^2 x$ $2\sin x \cos^2 x - \sin x = 0$ $(2\cos^2 x - 1) = 0$ $\cos^2 x = \frac{1}{2}$ $\cos x = \pm\sqrt{\frac{1}{2}}$ $x = \pm 45° + k360°$; $k \in Z$ **or** $x = \pm 135° + k360°$; $k \in Z$	$\frac{\sin x}{\cos x}$ $2\sin x \cos x$ $\sin x = 2\sin x \cos^2 x$ $(2\cos^2 x - 1) = 0$ $\cos 2x = 0$ $\therefore x = \pm 45° + k180°$; $k \in Z$ $\frac{\sin x}{\cos x}$ $2\sin x \cos x$ $\sin x = 2\sin x \cos^2 x$ $(2\cos^2 x - 1) = 0$ $\cos x = \pm\sqrt{\frac{1}{2}}$ $x = \pm 45° + k360°$; $k \in Z$ **or** $x = \pm 135° + k360°$; $k \in Z$

QUESTION 9

(a)	$\sin\left(\hat{C}-\hat{D}\right)=\sin\hat{C}.\cos\hat{D}-\cos\hat{C}.\sin\hat{D}$ $=\left(\frac{12}{13}\right)\left(\frac{3}{5}\right)-\left(\frac{5}{13}\right)\left(-\frac{4}{5}\right)$ $=\frac{56}{65}$	$=\sin\hat{C}.\cos\hat{D}-\cos\hat{C}.\sin\hat{D}$ $\left(\frac{12}{13}\right)$ $\left(\frac{3}{5}\right)$ $\left(\frac{5}{13}\right)$ $\left(-\frac{4}{5}\right)$ $=\frac{56}{65}$
(b)	$\cos(90°+60°).\cos 28°+\cos 60°.\cos 62°$ $-\sin 60°\sin 62°+\cos 60°\cos 62°$ $\cos(60°+62°)$ $\cos 122°$ $=\cos(180°-58°)$ $=-\cos 58°$ $=-k$ **Alternate:** $-\sin 60°\cos 28°+\cos 60°\sin 28°$ $=-\sin(60°-28°)$ $=-\sin 32°$ $=-\cos 58°$ $=-k$ **Alternate:** $-\cos 30°\cos 28°+\sin 30°\sin 28°$ $=-\cos(30°+28°)$ $=-\cos 58°$ $=-k$	$\cos(90°+60°)$ $\cos(60°+62°)$ $=\cos(180°-58°)$ $=-\cos 58°$ $=-k$

QUESTION 10

(a)	In ΔAEC $\frac{EC}{\sin 60°} = \frac{80}{\sin 45°}$ $EC = \frac{80\sin 60°}{\sin 45°}$ $EC \approx 98$ m In ΔEDC: $C\hat{E}D = 135°$ (adj $\angle$s on str line) $(CD)^2 = (53)^2 + (97,98)^2 - 2(53)(97,98) \times \cos 135°$ $CD = 140,54499...$ m $CD \approx 140,5$ m	$\frac{EC}{\sin 60°} = \frac{80}{\sin 45°}$ $EC = \frac{80\sin 60°}{\sin 45°}$ $EC \approx 98$ m $C\hat{E}D = 135°$ $(CD)^2 = (53)^2 + (97,98)^2$ $-2(53)(97,98) \times \cos 135°$ $CD = 140,5$ m
(b)	In ΔACB: $\tan 37° = \frac{BC}{AC}$ $BC = 80\tan 37°$ $BC = 60,284$ m Let M be the midpoint of BC: In ΔDMC: $MC = \frac{1}{2}BC$ $\therefore MC = 30,142$ m $\tan C\hat{D}M = \frac{MC}{CD}$ $\tan C\hat{D}M = \frac{30,142}{140,55}$ $C\hat{D}M \approx 12,1°$ The angle of elevation of M from D is $12,1°$.	In ΔACB: $\tan 37° = \frac{BC}{AC}$ $BC = 60,284$ m $\tan C\hat{D}M = \frac{MC}{CD}$ $MC = 30,142$ m $C\hat{D}M \approx 12,1°$

QUESTION 11

(a)	Circle with centre P: $x^2 - 6x + y^2 - 12y = -41$ $(x-3)^2 + (y-6)^2 = 4$ Centre: P(3;6) Radius: 2 units	$(x-3)^2 + (y-6)^2 = 4$ P(3;6) Radius: 2 units
(b)	Centre: $Q(9;3)$ Distance $PQ = \sqrt{(9-3)^2 + (3-6)^2}$ Distance $PQ = \sqrt{45}$ Distance $PQ = 3\sqrt{5}$ $\therefore 3\sqrt{5} - (2+2)$ $= 2,7$	$= \sqrt{(9-3)^2 + (3-6)^2}$ $= 3\sqrt{5}$ $\therefore 3\sqrt{5} - (2+2)$
(c)	Volume of block $= lbh - 2\times(\pi r^2 h)$ $= (20\times14\times10) - 2(\pi(4)(20))$ $= 2800 - 160\pi$ $\approx 2297,3$ units^3	$= lbh - 2\times(\pi r^2 h)$ $= 2800 - 160\pi$ $\approx 2297,3$ units^3

QUESTION 12

(a)	$\bar{x}=\dfrac{5a+5b}{10}$ $\bar{x}=\dfrac{a+b}{2}$	$\bar{x}=\dfrac{a+b}{2}$ (1)
(b)	$\sigma^2=\dfrac{5\left[a-\dfrac{a+b}{2}\right]^2+5\left[b-\dfrac{(a+b)}{2}\right]^2}{10}$ $\sigma^2=\dfrac{\left(\dfrac{a-b}{2}\right)^2+\left(\dfrac{b-a}{2}\right)^2}{2}$ $\sigma^2=\dfrac{\dfrac{(a-b)^2}{4}+\dfrac{(a-b)^2}{4}}{2}$ $\sigma^2=\dfrac{(a-b)^2}{4}$ $\sigma=\dfrac{(a-b)}{2}$	$\sigma^2=\dfrac{5\left[a-\dfrac{a+b}{2}\right]^2+5\left[b-\dfrac{(a+b)}{2}\right]^2}{10}$ $\sigma^2=\dfrac{\left(\dfrac{a-b}{2}\right)^2+\left(\dfrac{b-a}{2}\right)^2}{2}$ $\sigma=\dfrac{(a-b)}{2}$

QUESTION 13

(a)	Let: $\hat{O}_1 = 2x$ $\therefore \hat{D} = x$ ($\angle$ at centre = 2x) $\therefore \hat{A}_1 = x$ (alt $\angle$s; AC//BC) $\therefore \hat{C} = x$ ($\angle$ in same segment) In ΔCAE: $\hat{E}_1 = 180° - 2x$ (int $\angle$s of Δ) $\therefore \hat{E}_2 = 2x$ (adj $\angle$s on str. lines) Since $\therefore \hat{E}_2 = 2x = \hat{O}_1$ And these are subtended by AB, Then AEOB is cyclic ($\angle$s in same segment =)	$\hat{D} = x$ ($\angle$ at centre = 2x) $\therefore \hat{A}_1 = x$ (alt $\angle$s; AC//BC) $\hat{C} = x$ ($\angle$ in same seg) $\hat{E}_2 = 2x$ (adj $\angle$s on str. line) And these are subtended by AB, then AEOB is cyclic ($\angle$s in same segment =)
(b)	Let: $\hat{D}_1 = x$ $\therefore \hat{B}_2 = x$ ... equal chords subtend = $\angle$s Let: $\hat{E}_1 = y$ $\therefore \hat{B}_1 = y$ ... ext. $\angle$ of cyclic quad = int opp $\therefore \hat{A}_1 = y - x$ ... ext. $\angle s$ of Δ = sum int opp $\therefore \hat{B}_1 - \hat{B}_2 = \hat{A}_1$	$\therefore \hat{B}_2 = x$ equal chords subtend = $\angle$s $\therefore \hat{B}_1 = y$ ext. $\angle$ of cyclic quad = int opp $\therefore \hat{A}_1 = y - x$ ext. $\angle s$ of Δ = sum int opp $\therefore \hat{B}_1 - \hat{B}_2 = \hat{A}_1$
(c)	Draw a perp. From P to SQ Call perp. PU $\therefore UQ = 5 - 3$ $UQ = 2$ cm $\therefore PQ = 3 + 5$ $PQ = 8$ cm $(PU)^2 = (PQ)^2 - (UQ)^2$ pythag $(PU)^2 = (8)^2 - (2)^2$ pythag $PU = \sqrt{60}$ $PU = 7{,}7$ cm PU = RS (rectangle) $\therefore RS = 7{,}7$ cm	Draw a perp. From P to SQ $UQ = 2$ cm $PQ = 8$ cm $(PU)^2 = (8)^2 - (2)^2$ pythag $PU = \sqrt{60}$ PU = RS (rectangle)

QUESTION 14

(a)	In ΔBOC: $\hat{C} = 90° - \theta$ (Isos Δ; Radii; Int $\angle$s of Δ) In ΔOCF: $\therefore \hat{C} = \theta$ $\frac{CF}{8} = \cos\theta$ $CF = 8\cos\theta$ $OF = 8\sin\theta$ $\therefore P = 2 \times CF + 4 \times OF$ $\therefore P = 16\cos\theta + 32\sin\theta$	$\therefore \hat{C}_2 = \theta$ $CF = 8\cos\theta$ $OF = 8\sin\theta$ $\therefore P = 2 \times CF + 4 \times OF$
(b)	$P = 16\cos\theta + 32\sin\theta$ and $P = 16\sqrt{5}\sin(\theta + \alpha)$ $P = 16\sqrt{5}\sin\theta.\cos\alpha + 16\sqrt{5}\cos\theta.\sin\alpha$ $\therefore 16\sqrt{5}\sin\alpha = 16$ and $16\sqrt{5}\cos\alpha = 32$ $\alpha \approx 26{,}6°$	$16\sqrt{5}\sin\theta.\cos\alpha + 16\sqrt{5}\cos\theta.\sin\alpha$ $\therefore 16\sqrt{5}\sin\alpha = 16$ $16\sqrt{5}\cos\alpha = 32$ $\alpha \approx 26{,}6°$

Total: 150 marks