NASIONALE SENIOR SERTIFIKAAT-EKSAMEN
NOVEMBER 2020

WISKUNDE: VRAESTEL I

NASIENRIGLYNE

Tyd: 3 uur — 150 punte

Hierdie nasienriglyne word voorberei vir gebruik deur eksaminatore en hulpeksaminatore. Daar word van alle nasieners vereis om 'n standaardiserings-vergadering by te woon om te verseker dat die nasienriglyne konsekwent vertolk en toegepas word tydens die nasien van kandidate se skrifte.

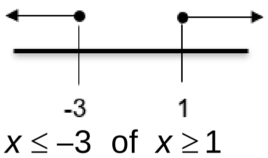
Die IEB sal geen gesprek aanknoop of korrespondensie voer oor enige nasienriglyne nie. Daar word toegegee dat verskillende menings rondom sake van beklemtoning of detail in sodanige riglyne mag voorkom. Dit is ook voor die hand liggend dat, sonder die voordeel van bywoning van 'n standaardiserings-vergadering, daar verskillende vertolkings mag wees oor die toepassing van die nasienriglyne.

NEEM KENNIS:

- Indien 'n kandidaat 'n vraag meer as een keer beantwoord, sien slegs die EERSTE poging na.
- Deurlopende akkuraatheid geld vir alle aspekte van die nasienmemorandum.

AFDELING A

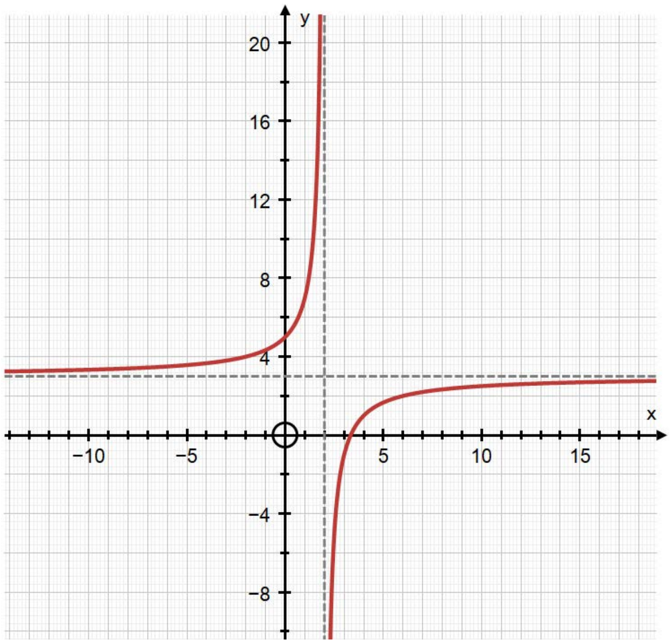
VRAAG 1

(a)(1)	$px^2+2x-3=0$ $x=\dfrac{(-2)\pm\sqrt{(2)^2-4(p)(-3)}}{2(p)}$ $x=\dfrac{-2\pm\sqrt{4+12p}}{2p}$ $x=\dfrac{-2\pm 2\sqrt{1+3p}}{2p}$ $x=\dfrac{-1\pm\sqrt{1+3p}}{p}$	gebruik kwadratiese formule $x=\dfrac{-2\pm\sqrt{4+12p}}{2p}$ vereenvoudigde oplossing
(a)(2)	Niereële wortels vir: $1+3p<0$ $p<-\dfrac{1}{3}$	$\Delta<0$ $p<-\dfrac{1}{3}$ Geen punte vir $\Delta>0$
(b)	$\sqrt{x-2}+4=x$ $(x-2)=(x-4)^2$ $x-2=x^2-8x+16$ $x^2-9x+18=0$ $x=6$ of $x=3$ nie geldig vir $x=3$	Isoleer wortelvorm x^2-4x+4 $x^2-8x+16$ faktore antwoord met seleksie
(c)	$(x+3)(x-1)\geq 0$ Kritieke waardes: -3 ; 1  $x\leq -3$ of $x\geq 1$	Getallelyn/grafiek $x\leq -3$ of $x\geq 1$

VRAAG 2

(a)	$x^{\frac{2}{3}} = 4$ $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(2^2\right)^{\frac{3}{2}}$ $x = \pm 8$ **Alternatief:** $\sqrt[3]{x^2} = 4$ $\left(\sqrt[3]{x^2}\right)^3 = (4)^3$ $x^2 = 64$ $x = 8$ of $x = -8$	$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(2^2\right)^{\frac{3}{2}}$ $x = 8$ $x = -8$ $\left(\sqrt[3]{x^2}\right)^3 = (4)^3$ $x = 8$ $x = -8$
(b)	$x^2 + 1 = x - y$ Vervang: $y = 2 - 3x$ $x^2 + 1 = 4x - 2$ $x^2 - 4x + 3 = 0$ $x = 1$ of $x = 3$ Wanneer $x = 1$; $y = -1$ Wanneer $x = 3$; $y = -7$ **Alternatief:** $y = 2 - 3x$ ... verg. 1 $3^{x^2+1} = 3^{x-y}$ ... vervang verg. 1 $3^{x^2+1} = 3^{x-(2-3x)}$ $3^{x^2+1} = 3^{4x-2}$ $x^2 + 1 = 4x - 2$ $x^2 - 4x + 3 = 0$ $x = 1$ of $x = 3$ Wanneer $x = 1$; $y = -1$ Wanneer $x = 3$; $y = -7$	$x^2 + 1 = x - y$ Vervang: $y = 2 - 3x$ $x^2 + 1 = 4x - 2$ $x = 1$ $y = -1$ $x = 3$ $y = -7$ $3^{x^2+1} = \frac{3^x}{3^y}$ ... vervang verg. 1 $3^{x^2+1} = 3^{4x-2}$ $x^2 + 1 = 4x - 2$ $x = 1$ $y = -1$ $x = 3$ $y = -7$
(c)	$A = P(1+i)^n$ $25\,000 = 20\,000\left(1 + \frac{4}{100}\right)^n$ $\frac{5}{4} = (1{,}04)^n$ $n = \log_{1{,}04}\left(\frac{5}{4}\right)$ $n \approx 5{,}7$ jaar Na 6 jaar	$25\,000 = 20\,000\left(1 + \frac{4}{100}\right)^n$ $n = \log_{1{,}04}\left(\frac{5}{4}\right)$ $n \approx 5{,}7$ jaar 6 jaar

VRAAG 3

(a)	$f(0)=3-\dfrac{4}{0-2}$ $f(0)=5$	$f(0)=5$
(b)	$3-\dfrac{4}{x-2}=0$ $3(x-2)-4=0$ beperking $x \neq 2$ $3x-6-4=0$ $x=\dfrac{10}{3}$ $x=3\dfrac{1}{3}$	$3(x-2)-4=0$ $x=3\dfrac{1}{3}$
(c)		Vorm Vertikale asimptoot Horisontale asimptoot Afsnitte
(d)(1)	$f(x+p)=3-\dfrac{4}{x+p-2}$ $f(x+p)=-\dfrac{4}{[x+(p-2)]}+3$	$f(x+p)=3-\dfrac{4}{x+p-2}$
(d)(2)	Grafiek van *f* sal *p* eenhede na regs skuif	Verduideliking

(e)(1)	Vir $f^{-1}(x)$: $x = 3 - \dfrac{4}{y-2}$ $x = 3 - \dfrac{4}{y-2}$ $\dfrac{4}{y-2} = 3 - x$ $4 = (3-x)(y-2)$ $4 = 3y - 6 - xy + 2x$ $3y - xy = 4 + 6 - 2x$ $y(3-x) = 10 - 2x$ $y = \dfrac{10-2x}{3-x}$ $\therefore f^{-1}(x) = \dfrac{10-2x}{3-x}$ **Alternatiewe finale antwoord:** $f^{-1}(x) = \dfrac{2x-10}{x-3}$	$x = 3 - \dfrac{4}{y-2}$ $4 = (3-x)(y-2)$ $\therefore f^{-1}(x) = \dfrac{10-2x}{3-x}$ **Alternatiewe finale antwoord:** $f^{-1}(x) = \dfrac{2x-10}{x-3}$ **Alternatiewe finale antwoord:** $y = -\dfrac{4}{x-3} + 2$
(e)(2)	Definisiegebied van $f^{-1}(x)$: $x \in R$; $x \neq 3$	$x \in R$; $x \neq 3$

VRAAG 4

(4)(a)	$ar^2 = 7$ $ar^5 = -2\,401$ $\therefore \frac{ar^5}{ar^2} = -\frac{2\,401}{7}$ $\therefore r^3 = -343$ $\therefore r = -7$ $T_n = a(-7)^{n-1}$ $T_3 = a(-7)^{3-1} = 7$ $a = \frac{7}{49}$ $\therefore a = \frac{1}{7}$	$ar^2 = 7$ $ar^5 = -2\,401$ $\therefore \frac{ar^5}{ar^2} = -\frac{2\,401}{7}$ $r = -7$ $a = \frac{1}{7}$
(4)(b)(1)	3 7 15 27 Ry 4 8 12 Eerste verskil 4 4 Konstante tweede verskil	Ry Eerste verskil Konstante tweede verskil
(b)(2)	$2a = 4 \quad \therefore a = 2$ $3a + b = 4 \quad \therefore \quad b = -2$ $a + b + c = 3 \quad \therefore \quad c = 3$ $T_n = 2n^2 - 2n + 3$ **Alternatief:** $T_n = 7(n-1) - 3(n-2) + \frac{(n-1)(n-2)}{2} \times (4)$ $T_n = 7n - 7 - 3n + 6 + (n^2 - 3n + 2)(2)$ $T_n = 2n^2 - 2n + 3$	Metode $a = 2$ $b = -2$ $c = 3$ Metode $a = 2$ $b = -2$ $c = 3$

VRAAG 5

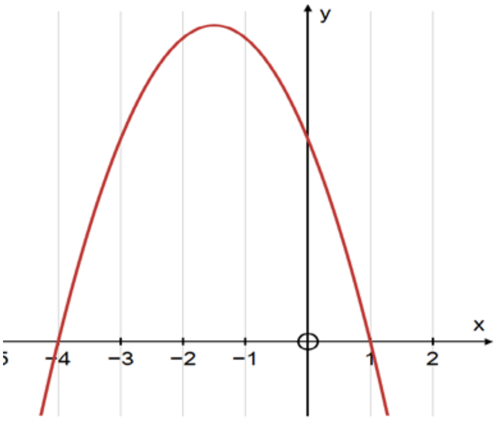
(a)	$g(x) = \log_t x$ vervang $(2;-1)$ $-1 = \log_t 2$ $t^{-1} = 2$ $t = \frac{1}{2}$	$-1 = \log_t 2$ $t = \frac{1}{2}$
(b)	X-afsnit van normale/standaard log-grafiek is altyd: $(1;0)$ aangesien $\log_t 1 = 0$ $\therefore C(1\,;\,0)$ **Alternatief:** Vir koördinate van C: X-afsnit, laat $y = 0$. $y = \log_{\frac{1}{2}} x$ $0 = \log_{\frac{1}{2}} x$ $x = \left(\frac{1}{2}\right)^0$ $x = 1$ $\therefore C(1;0)$	$\therefore C(1;0)$ $\therefore C(1;0)$
(c)	$f(x) = 2p^x + q$ $q = -1$ aangesien asimptoot deur $A(2;-1)$ gaan $f(x) = 2p^x - 1$ ... vervang $(1;0)$ $0 = 2p^1 - 1$ $\therefore p = \frac{1}{2}$	$q = -1$ $0 = 2p^1 - 1$ $p = \frac{1}{2}$
(d)	D is die y-afsnit van f : Laat $x = 0$. $f(x) = 2 \times \left(\frac{1}{2}\right)^x - 1$ ... vervang $x = 0$ $y = 2 \times \left(\frac{1}{2}\right)^0 - 1$ $y = 1$ $\therefore D(0\,;1)$	$y = 2 \times \left(\frac{1}{2}\right)^0 - 1$ $D(0;1)$
(e)	$f(x) = 2\left(\frac{1}{2}\right)^x - 1$ ... vervang $B(2;y)$ $f(x) = 2\left(\frac{1}{2}\right)^2 - 1$ $f(x) = y = -\frac{1}{2}$ Lengte van AB $= \frac{1}{2}$	$f(x) = 2\left(\frac{1}{2}\right)^2 - 1$ Lengte van AB $= \frac{1}{2}$
(f)	Waardegebied van f : $y > -1$	$y > -1$

VRAAG 6

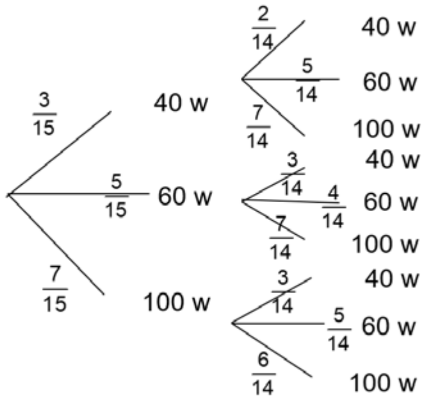
(a)	$f(x)=1-2x+x^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{1-2(x+h)+(x+h)^2-(1-2x+x^2)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{1-2x-2h+x^2+2xh+h^2-1+2x-x^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-2h+2xh+h^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{h(-2+2x+h)}{h}$ $f'(x)=\lim\limits_{h\to 0}(-2+2x+h)$ $2x-2$	$1-2(x+h)+(x+h)^2$ $-(1-2x+x^2)$ Kwadrering en verspreiding Faktorisering Notasie Vervang om $2x-2$ te kry
(b)	$y=x^{10}+10x$ $\dfrac{dy}{dx}=10x^9+10$	$10x^9$ 10
(c)	$y=\dfrac{5}{x^3}+\dfrac{x^{\frac{1}{2}}}{x^3}$ $y=5x^{-3}+x^{-\frac{5}{2}}$ $\dfrac{dy}{dx}=-15x^{-4}-\dfrac{5}{2}x^{-\frac{7}{2}}$	$y=5x^{-3}+x^{-\frac{5}{2}}$ $\dfrac{dy}{dx}=-15x^{-4}-\dfrac{5}{2}x^{-\frac{7}{2}}$ Penaliseer 1 vir notasie

AFDELING B

VRAAG 7

(a)	Vir: $x < -4$ en $x > 1$	$x < -4$ $x > 1$
(b)		Vorm X-afsnitte
(c)	$k > p$ of $k < q$	$k > p$ $k < q$
(d)	$x > -1\frac{1}{2}$	$x > -1\frac{1}{2}$

VRAAG 8

(a)(1)	8^6	8^6
(a)(2)	$= 20\,160$	$8 \times 7 \times 6 \times 5 \times 4 \times 3$ 20 160
(b)(1)		$\frac{3}{15}$; $\frac{5}{15}$ en $\frac{7}{15}$ $\frac{\square}{14}$ Takke met korrekte waardes
(b)(2)	$\left(\frac{5}{15} \times \frac{7}{14}\right) + \left(\frac{7}{15} \times \frac{5}{14}\right)$ $= \frac{1}{3}$	$\left(\frac{5}{15} \times \frac{7}{14}\right)$ $\left(\frac{7}{15} \times \frac{5}{14}\right)$ $= \frac{1}{3}$
(c)	$P(A \cap B) = P(A) \times P(B)$ $\therefore P(A \cap B) = 0{,}08 \times 0{,}02$ $= 0{,}0016$ maar $P(AUB) = P(A) + P(B) - P(A \cap B)$ $\therefore P(AUB) = 0{,}08 + 0{,}02 - 0{,}0016$ $= 0{,}0984$ **Alternatief:** P(minstens een wen) $=$ P(een of meer wenne) $= 1 -$ P(geen wenne) $= 1 - P(L) \times P(L)$ $= 1 - 0{,}98 \times 0{,}92$ $= 0{,}0984$	$\therefore P(A \cap B) = 0{,}0016$ $P(AUB) = P(A) + P(B) -$ $P(A \cap B)$ $\therefore P(AUB) = 0{,}08 + 0{,}02 - ...$ $= 0{,}0984$ $= 1 -$ P(geen wenne) 0,98 0,92 $= 0{,}0984$

VRAAG 9

(a)	$a=725$ $b=190$	$a=725$ $b=190$
(b)	$h=k(x-a)^2+b$ $h=k(x-725)^2+190$ vervang $(0;315)$ $315=k(0-725)^2+190$ $k=\frac{1}{4\,205}$ $h=\frac{1}{4\,205}(x-725)^2+190$ ... vervang $(x;210)$ $210=\frac{1}{4\,205}(x-725)^2+190$ $x=1015$ of $x=435$ Dus is die horisontale afstand van higrometer van die linkertoring af 435 m.	$h=k(x-725)^2+190$ $k=\frac{1}{4\,205}$ $210=\frac{1}{4\,205}(x-725)^2$ $+190$ $x=1015$ of $x=435$ Dus is die horisontale afstand van higrometer van die linkertoring af 435 m.

VRAAG 10

(a)	$F = 8\,755\left[\dfrac{\left(1+\dfrac{6,7}{400}\right)^{(5\times 4)}-1}{\dfrac{6,7}{400}}\right]$	Binne vierkante hakie Korrekte x in korrekte formule
	$F = 205\,973,485$	$F = 205\,973,485$
	Totale koste van aandele $= 8\,755 \times 4 \times 5$ Totale koste van aandele $= 175\,100$	$175\,100$
	Totale wins $= 30\,873,485$	$30\,873,485$
	% wins $= \dfrac{30\,873,485}{175\,100} \times 100$ $= 17,6319\,\%$ $\approx 17,6\%$	$\approx 17,6\%$ %
	Alternatief: $F = 8\,755\left[\dfrac{\left(1+\dfrac{6,7}{400}\right)^{(5\times 4)}-1}{\dfrac{6,7}{400}}\right]$	Binne vierkante hakie Korrekte x in korrekte formule
	$F = 205\,973,485$	$F = 205\,973,485$
	Totale koste van aandele $= 8\,755 \times 4 \times 5$ Totale koste van aandele $= 175\,100$	$175\,100$
	$\therefore$ % wins $= \dfrac{205973,485}{175100}$ $= 1,176319$ $\therefore 17,6\%$	% wins $= \dfrac{205973,485}{175100}$ $\approx 17,6\%$ %

(b)	$300\,000 = x\left[\dfrac{1-\left(1+\dfrac{9,5}{1200}\right)^{-(15\times 12)}}{\dfrac{9,5}{1200}}\right]$ $x = 3\,132,674049$ Saldo van lening $= A - F$ $A = 300\,000\left(1+\dfrac{9,5}{1200}\right)^{12\times 5}$ $A = 481\,502,8408$ $F = 3\,132,674049\left[\dfrac{\left(1+\dfrac{9,5}{1200}\right)^{(12\times 5)}-1}{\dfrac{9,5}{1200}}\right]$ $F = 239\,405,9954$ Saldo van lening $= (481\,502,8408) - (239\,405,9954)$ $= 242\,096,8454$ $\approx 242\,096,85$ **Alternatief:** $P = 3\,132,674049\left[\dfrac{1-\left(1+\dfrac{9,5}{1200}\right)^{-(10\times 12)}}{\dfrac{9,5}{1200}}\right]$ $P = 242\,096,8454$ $\approx 242\,096,85$ Nee, daar sal 'n tekort van $R36\,123,36$ wees.	300 000 Binne die vierkante hakie $x = 3132,674049$ $F = 239\,405,9954$ $= (481\,502,8408) - (239\,405,9954)$ Comparison between 10a and 10b with conclusion …………………………………… 300 000 Inside the square bracket $x = 3\,132,674049$ $P = 3\,132,674049\left[\dfrac{1-\left(1+\dfrac{9,5}{1200}\right)^{-(10\times 12)}}{\dfrac{9,5}{1200}}\right]$ No. of years: -120 $\approx 242\,096,85$ Comparison between 10a and 10b with conclusion

VRAAG 11

(a)	$\sum_{i=1}^{\infty}\frac{k}{2i}+\sum_{i=1}^{10}2^{2i}>1000\,000$ Werk met: $\sum_{i=1}^{\infty}\frac{k}{2^i}$ $T_1=\frac{k}{2}$; $T_2=\frac{k}{4}$; $T_3=\frac{k}{8}$ Gemene verhouding: $\frac{k}{4}\div\frac{k}{2}$ $r=\frac{1}{2}$ $S_\infty=\frac{a}{1-r}$ vir $-1<r<1$ $S_\infty=\frac{\frac{k}{2}}{1-\frac{1}{2}}$ $S_\infty=k$ Werk met: $\sum_{i=1}^{10}2^{2i}$ $T_1=2^2$; $T_2=2^4$; $T_3=2^6$ Gemene verhouding: $r=4$ $S_n=\frac{a\left(r^n-1\right)}{r-1}$; $r\neq1$ $S_{10}=\frac{4\left(4^{10}-1\right)}{4-1}$ $S_{10}=1\,398\,100$ $\therefore\sum_{i=1}^{\infty}\frac{k}{2i}+\sum_{i=1}^{10}2^{2i}>1\,000\,000$ kan herskryf word as $k+1\,398\,100>1000\,000$ $k>-398\,100$ $\therefore k=-398\,099 \quad (k\in\mathbb{Z})$	$r=\frac{1}{2}$ Korrekte vervanging in korrekte formule om te kry: $S_\infty=k$ $r=4$ Korrekte vervanging in korrekte formule om te kry: $S_{10}=1\,398\,100$ $k+1\,398\,100>1000\,000$ $\therefore k=-398\,099 \quad (k\in\mathbb{Z})$
(b)(1)	$5+\frac{15}{2}+10+...+\frac{505}{2}$ Gemene verskil van $\frac{5}{2}$; reeks is rekenkundig $T_n=a+(n-1)d$ $\frac{505}{2}=5+(n-1)\left(\frac{5}{2}\right)$ $250=\frac{5}{2}n$ $n=100$	$\text{d}=\frac{5}{2}$ Korrekte vervanging in die korrekte formule $n=100$

(b)(2)	Middelste 30 terme sal wees: T_{36} tot T_{65} $T_{36}=5+(35)\left(\frac{5}{2}\right)$ $T_{36}=\frac{185}{2}$ Laat $a=\frac{185}{2}$; $d=\frac{5}{2}$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{30}=\frac{30}{2}\left[2\left(\frac{185}{2}\right)+(29)\left(\frac{5}{2}\right)\right]$ $S_{30}=3\ 862{,}5$ **Alternatief:** Middelste 30 terme sal wees: T_{36} tot T_{65} $T_{36}=5+(35)\left(\frac{5}{2}\right)$ $T_{36}=\frac{185}{2}$ $T_{65}=5+(64)\left(\frac{5}{2}\right)$ $T_{65}=165$ $S_n=\frac{n}{2}(a+l)$ $S_{30}=\frac{30}{2}\left(\frac{185}{2}+165\right)$ $S_{30}=3\ 862{,}5$	T_{36} $T_{36}=\frac{185}{2}$ Korrekte vervanging in korrekte formule $S_{30}=3\ 862{,}5$ T_{36} $T_{36}=\frac{185}{2}$ Korrekte vervanging in korrekte formule $S_{30}=3\ 862{,}5$

VRAAG 12

12	Laat $g(1)=h(1)$. $(1)^3-a(1)^2+6=2(1)^2+b(1)+3$ $1-a+6=2+b+3$ $a=2-b$ ... verg. 1 $g'(x)=3x^2-2ax$ $h'(x)=4x+b$ $g'(1)=h'(1)$ $3(1)^2-2a(1)=4(1)+b$ $3-2a=4+b$ ... vervang verg. 1: $a=2-b$ $3-2(2-b)=4+b$ $b=5$ $a=-3$ $h(x)=2x^2+5x+3$ $h(1)=10$ Kontakpunt is: $(1;10)$	$g(1)=h(1)$ $a=2-b$ ... verg.1 $g'(x)=3x^2-2ax$ $h'(x)=4x+b$ $g'(1)=h'(1)$ $2a+b=-1$ $b=5$ $(1;10)$

VRAAG 13

13	$8x + 4x + 4h = P$ $P = 12x + 4h$ $P - 12x = 4h$ $\therefore h = \frac{1}{4}P - 3x$ $V = l \times b \times h$ $V = (2x)(x)(h)$ ... vervang: $h = \frac{1}{4}P - 3x$ $V = (2x)(x)\left(\frac{1}{4}P - 3x\right)$ $V = \frac{1}{2}x^2P - 6x^3$ $V' = Px - 18x^2$ $0 = x(P - 18x)$ $x = 0$ of $x = \frac{P}{18}$ Dus is lengte van die boks $2x = \frac{P}{9}$. $\therefore$ lengte van boks is $\frac{1}{9}P$ cm wanneer die volume 'n maksimum is.	$8x + 4x + 4h = P$ $h = \frac{1}{4}P - 3x$ $V = (2x)(x)\left(\frac{1}{4}P - 3x\right)$ $V = \frac{1}{2}x^2P - 6x^3$ $V' = Px - 18x^2$ $0 = x(P - 18x)$ $P = 18x$ Lengte van die boks is $2x$ en $P = 18x$

Totaal: 150 punte