NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2018

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a)
$$T_{100} = a + 99d$$
$$a + 99(7) = 512$$
$$a = -181$$

(b) (1) $T_1 = 2(1) + 3 \therefore T_1 = 5$; $T_2 = 7$; $T_3 = 9$

$\therefore$ Constant first difference $= 2$

(2)
$$S_n = \frac{n}{2}\left[2(5) + (n-1)(2)\right]$$
$$S_n = \frac{n}{2}[8 + 2n]$$
$$S_n = 4n + n^2$$

Alternate:
$$S_n = \frac{n}{2}(a + l)$$
$$S_n = \frac{n}{2}(5 + 2n + 3)$$
$$S_n = n^2 + 4n$$

(c)
$2a = 4 \qquad \therefore a = 2$

$3a + b = 3 \qquad \therefore 3(2) + b = 3 \qquad \therefore b = -3$

$a + b + c = 4 \qquad \therefore 2 + (-3) + c = 4 \qquad \therefore c = 5$

$T_n = 2n^2 - 3n + 5$

QUESTION 2

(a) (1)
$$T_1 = 108 \times \left(\frac{2}{3}\right)^1 \qquad \therefore T_1 = 72$$
$$T_2 = 108 \times \left(\frac{2}{3}\right)^2 \qquad \therefore T_2 = 48$$

(2)
$$T_3 = 108 \times \left(\frac{2}{3}\right)^3 \qquad \therefore T_3 = 32$$
$$T_4 = 108 \times \left(\frac{2}{3}\right)^4 \qquad \therefore T_4 = \frac{64}{3}$$

$\therefore$ First 4 items add up to $\frac{520}{3}$

$\therefore x = 4$

Alternative:

Geometric sequence with $a = 72$ and $r = \frac{2}{3}$

$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$

$\left(\frac{2}{3}\right)^n = \frac{16}{81}$

$\log_{\frac{2}{3}}\left(\frac{16}{81}\right) = n$

$n = 4$

$\therefore x = 4$

(b) Area $1 = 2\pi(21)^2$

Area $2 = 2\pi(3)^2$

Area $3 = 2\pi\left(\frac{3}{7}\right)^2$

Common ratio: $\frac{1}{49}$ indicating a convergent series

$S\infty = \frac{a}{1-r}$; $-1 < r < 1$

$S\infty = \frac{2\pi(21)^2}{1-\frac{1}{49}}$

$S\infty = \frac{7\ 203}{8}\pi$ $\therefore S\infty \approx 2\ 828{,}6\ \text{cm}^2$

QUESTION 3

(a) (1) Working with: $\frac{1}{\left(x^2 - 3x - 4\right)\left(x + 1\right)}$, undefined for:

$\left(x^2 - 3x - 4\right)\left(x + 1\right) = 0$

$\left(x - 4\right)\left(x + 1\right)\left(x + 1\right) = 0$

$x = 4$ or $x = -1$

(2) $x^2 - 3x - 4 \leq 0$

Critical values: 4 ; -1

$\therefore -1 \leq x \leq 4$

(b) (1) $x + 4 \geq 0$

$\therefore x \geq -4$

(2) $\sqrt{x+4}-3=x$

$\left(\sqrt{x+4}\right)^2=(x+3)^2$

$x+4=x^2+6x+9$

$x^2+5x+5=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x\approx-1{,}4$ or $x\approx-3{,}6$ (n/v)

QUESTION 4

(a) (1) Average Gradient $=\dfrac{[2(1+h)^3]-[2(1)^3]}{(1+h)-1}$

Average Gradient $=\dfrac{2(1+h)(1+2h+h^2)-2}{h}$

Average Gradient $=\dfrac{2(1+2h+h^2+h+2h^2+h^3)-2}{h}$

Average Gradient $=\dfrac{2(1+3h+3h^2+h^3)-2}{h}$

Average Gradient $=\dfrac{(2+6h+6h^2+2h^3)-2}{h}$

Average Gradient $=\dfrac{h(6+6h+2h^2)}{h}$

Average Gradient $=6+6h+2h^2$

(2) $f'(1)=\lim\limits_{h\to0}\left(6+6h+2h^2\right)$

$f'(1)=6$

Alternate:

(2) $f'(x)=\lim\limits_{h\to0}\dfrac{2(x+h)^3-2x^3}{h}$

$f'(x)=\lim\limits_{h\to0}\dfrac{2(x+h)(x^2+2xh+h^2)-2x^3}{h}$

$f'(x)=\lim\limits_{h\to0}\dfrac{2(x^3+2x^2h+h^2x+x^2h+2xh^2+h^3)-2x^3}{h}$

$f'(x)=\lim\limits_{h\to0}\dfrac{6x^2h+6h^2x+2h^3}{h}$

$f'(x)=\lim\limits_{h\to0}\dfrac{h(6x^2+6hx+2h^2)}{h}$

$f'(x)=6x^2$

$f'(1)=6(1)^2 \quad \therefore f'(1)=6$

Alternate:

$$f(x) = 2x^3$$
$$f'(x) = 6x^2$$
$$f'(1) = 6$$

(b) $y = 3x^{-2} - 10x^{\frac{1}{5}}$

$$\frac{dy}{dx} = -6x^{-3} \ -2x^{-\frac{4}{5}}$$

QUESTION 5

(a) $A = 300000\left(1+\frac{0,16}{12}\right)^{60}\left(1+0,11\right)^{10} - 500000\left(1+0,11\right)^2$

$A = 1269728,917$

Alternate:

$T_0 - T_5$: $A = 300\ 000\left(1+\frac{16}{100(12)}\right)^{5\times 12}$

$A = 664\ 142,0648$

$T_6 - T_{13}$: $\text{A} = 664\ 142,0648\left(1+\frac{11}{100}\right)^8$

At the end of the 13th year: $1\,530\,540,473 - 500\ 000$

$T_{14} - T_{15}$: $A = 1\,030\ 540,473\left(1+\frac{11}{100}\right)^2$

At the end of the 15th year he has: R1 269 728,917

(b) $F = x\left[\frac{(1+n)^n - 1}{i}\right]$

$$1\,270\,000 = x\left[\frac{\left(1+\frac{8}{100(12)}\right)^{(15\times 12)} - 1}{\frac{8}{100(12)}}\right]$$

$x = \text{R3 } 670,114804$

QUESTION 6

(a) Y-intercept: $y = 2(0) + 5$ $\therefore$ y-intercept for both graphs: (0 ; 5)

For horizontal asymptote for *f* :substitute $(-1\,;\,y)$ in $g(x) = 2x + 5$

$\therefore g(-1) = 2(-1) + 5$ $\therefore g(-1) = 3$

$\therefore$ Horizontal asymptote of *f*: $y = 3$

$f(x) = \dfrac{a}{x+1} + 3$ substitute (0 ; 5)

$5 = \dfrac{a}{0+1} + 3$ $\therefore a = 2$

$a = 2$; $b = 1$ and $c = 3$

(b) (1) X-intercept of f : $0 = \dfrac{2}{x+1} + 3$ $\therefore x = -\dfrac{5}{3}$

X-intercept of g : $0 = 2x + 5$ $\therefore x = -\dfrac{5}{2}$

(2) $-\dfrac{5}{3} \le x < -1$ or $x \le -\dfrac{5}{2}$

(c) (1) $g(x) = 2x + 5$

$x = 2y + 5$

$y = \dfrac{1}{2}x - \dfrac{5}{2}$

(2) Point of intersection: $2x + 5 = \dfrac{x-5}{2}$ $\therefore x = -5$

The values of *x* for which $g^{-1}(x) > g(x)$: $x < -5$

SECTION B

QUESTION 7

(a) $x = 5 \pm \sqrt{2}$

$\therefore \left[x - \left(5 + \sqrt{2}\right)\right]\left[x - \left(5 - \sqrt{2}\right)\right] = 0$

$x^2 - 5x + \sqrt{2}x - 5x - \sqrt{2}x + 23 = 0$

$x^2 - 10x + 23 = 0$

(b) For real and equal roots: Quadratic must be a perfect square $\therefore$

$x^2 + ax + b = 0$

$\left(x + \sqrt{b}\right)^2 = 0$

$x^2 + 2\sqrt{b}x + b = 0$

$\therefore a = 2\sqrt{b}$

$\therefore \left(\sqrt{b}\right)^2 = \left(\frac{a}{2}\right)^2$

$\therefore b = \frac{a^2}{4}$ … eq. 1

$x^2 + bx + a = 0$

$\left(x + \sqrt{a}\right)^2 = 0$

$x^2 + 2\sqrt{a}x + a = 0$

$\therefore b = 2\sqrt{a}$ … eq. 2

Substitute eq1 in eq 2:

$\frac{a^2}{4} = 2\sqrt{a}$

$\therefore a^{\frac{3}{2}} = 2^3$

$\therefore \left(a^{\frac{3}{2}}\right)^{\frac{2}{3}} = \left(2^3\right)^{\frac{2}{3}}$

$\therefore a = 4$ and $b = 4$

Alternate:

For real and equal roots, $\Delta = b^2 - 4ac = 0$

For $x^2 + ax + b = 0$: $\quad 0 = a^2 - 4b$

$\therefore b = \dfrac{a^2}{4}$ … eq1

For $x^2 + bx + a = 0$: $\quad 0 = b^2 - 4a$ … eq2

Substitute eq1 in eq 2:

$\left(\dfrac{a^2}{4}\right)^2 - 4a = 0$

$a^4 - 64a = 0$

$a(a^3 - 64) = 0$

$a = 0$ or $a = 4$

$\therefore a = 4$ only and $b = 4$

QUESTION 8

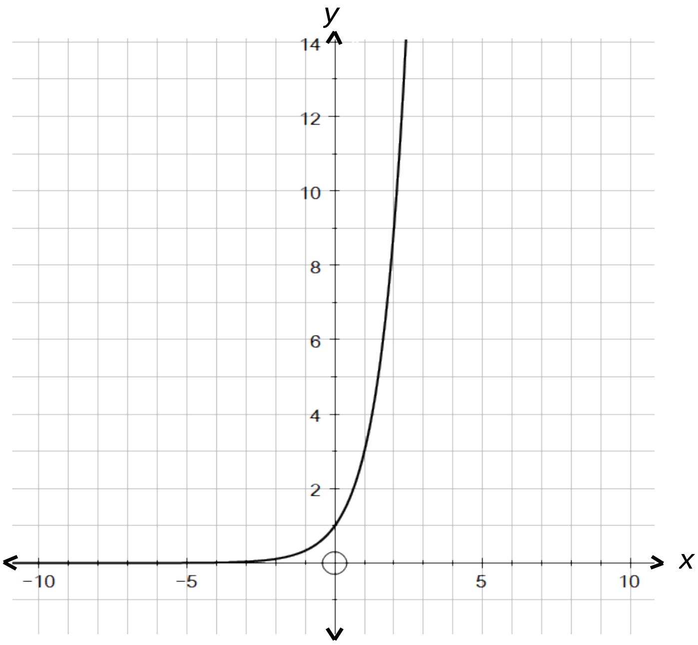
(a) $A = P(1+i)^n$

$y^2 = y(1+i)^x$

$y^2 = y\left(1+\frac{200}{100}\right)^x$

$y = (3)^x$

(b)



Shape
Y-intercept
Asymptote

(c) (1) $750 = (3)^x$

$x = \log_3 750$

$x \approx 6{,}03$

It took approximately 6 years

(2) Domain: $x > 6$ (accept: $x \geq 6$)

QUESTION 9

(a) For point of inflection: Let $g''(x)=0$

$g'(x)=3x^2-6x$
$g''(x)=6x-6$
$6x-6=0 \quad \therefore x=1$

$g(1)=-2$ and $h(1)=-2$
Hence, g and h intersect at $x=1$, the point of inflection.

Alternate:

For point of inflection: Let $g''(x)=0$
$g'(x)=3x^2-6x$
$g''(x)=6x-6$
$6x-6=0 \quad \therefore x=1$

Point of intersection: $x^3-3x^2=-\frac{2}{3}x-\frac{4}{3}$
$3x^3-9x^2+2x+4=0$
$x=\frac{3\pm\sqrt{21}}{3}$ or $x=1$
Therefore, the graph of h does intersect the graph of g at its point of inflection.

Alternate:

For point of inflection: Let $g''(x)=0$
$g'(x)=3x^2-6x$
$g''(x)=6x-6$
$6x-6=0 \quad \therefore x=1$
For co-ordinate of point of inflection:
Substitute $x=1$ in $f(1)=(1)^3-3(1)^2$

$f(1)=-2$

Substitute $(1;-2)$ in $y=-\frac{2}{3}x-\frac{4}{3}$

$\text{RHS}=-\frac{2}{3}(1)-\frac{4}{3}$
$\text{RHS}=-2$

$\text{RHS}=\text{LHS}$

Therefore, the graph of h does intersect the graph of g at its point of inflection.

(b) (1) For stationary point of $y = g'(x)$

$y = 3x^2 - 6x$

$\frac{dy}{dx} = 6x - 6$

$6x - 6 = 0$

$\therefore x = 1$

Startionary point $(1;-3)$ Min. value function

(2) (i) Concave down for: $x < 1$

(ii) $g'(1) = 3(1)^2 - 6(1)$

$g'(1) = -3$

(3) Decreasing gradient occurs for: $0 < x < 2$

Maximum decreasing gradient occurs at the point of inflection.

(c) The graph of g decreases for the interval: $0 < x < 2$

We must shift the graph of g , 3 units to the left

$\therefore k = 3$

QUESTION 10

(a) (1) Since $b > 2a$, then $b^2 > 4a^2$

Since $c < a$

Then $b^2 > 4ac$

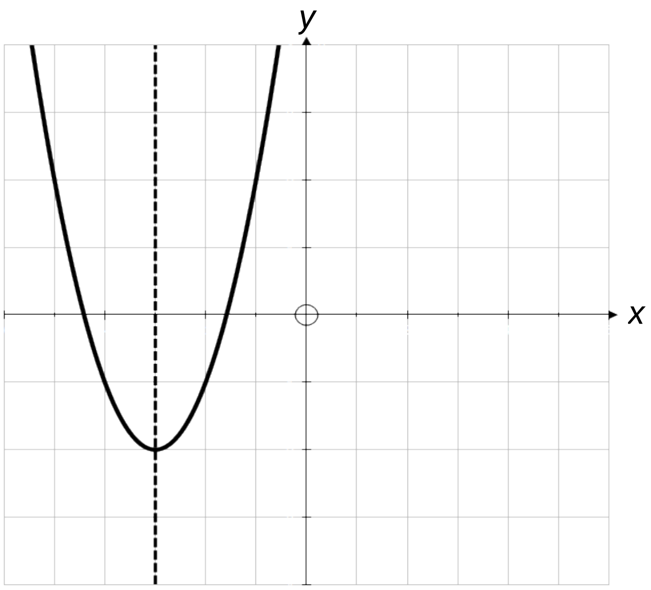
Alternate:

$b > 2a$ and $b > c$

$(b > 2a > a > c)$ **,** hence

$b^2 > 4ac$

(2)



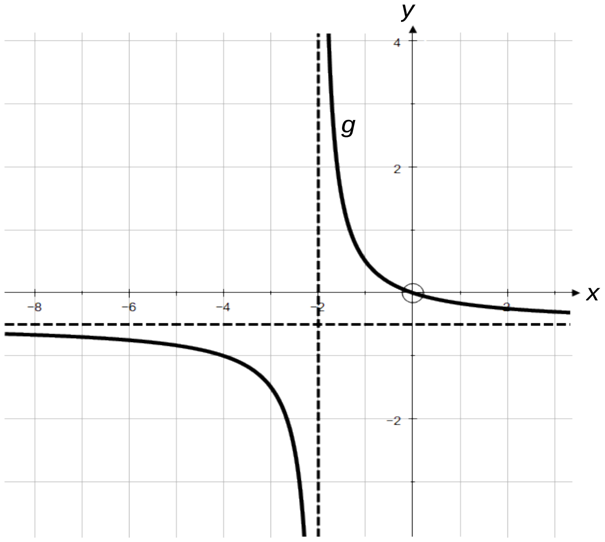
From the given constraints:

a , b and c are positive(+)

Therefore:

Shape: min. value function
Y-int. : +
Axis of Symm.: $x =$ negative value
$b^2 - 4ac > 0 \ \therefore$ roots are real and unequal

(b) (1)



Shape and Pt.$(0;0)$

Horizontal asymptote: $y = -\frac{1}{2}$

Vertical asymptote: $x = -2$

(2) $p \geq -\frac{1}{2}$

QUESTION 11

(a) (1) $P(\text{both letters are C}) = \frac{2}{6} \times \frac{1}{5}$

$= \frac{1}{15}$

(2) $P(\text{only one letter is C}) \left(\frac{2}{6} \times \frac{4}{5}\right) + \left(\frac{4}{6} \times \frac{2}{5}\right)$

$= \frac{8}{15}$

(b) $\frac{6!}{2!} = 360$

(c) $4! = 24$

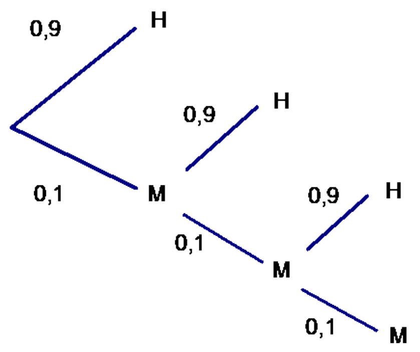
QUESTION 12

Let the number of missiles required for firing be *n*.
P(all will miss) $= (1-0,9)^n$ $\therefore$ P(all will miss) $= 0,1^n$
P(at least 1 will hit) $= 1-0,1^n$
We require: $1-0,1^n > 0,97$
When $n=1$, $1-0,1^1 = 0,9$
When $n=2$, $1-0,1^2 = 0,99$
When $n=3$, $1-0,1^3 = 0,999$
Therefore, at least 2 missiles should be fired.
Therefore, Lulu was correct.

Alternate:

Let the number of missiles required for firing be *n*.
P(all will miss) $= (1-0,9)^n$ $\therefore$ P(all will miss) $= 0,1^n$
Let: $1-0,1^n = 0,97$
$0,03 = 0,1^n$
$\log_{0,1} 0,03 = n$
$n \approx 1,5$
Therefore, at least 2 missiles need to be fired to ensure at least a 0,97 chance of hitting the target.
Lulu was correct

Alternate:



First missile fired: P(a hit) = 0,9

Second missile fired: P(a hit) = 0,9 + MH

= 0,9 + (0,1 x 0,9)

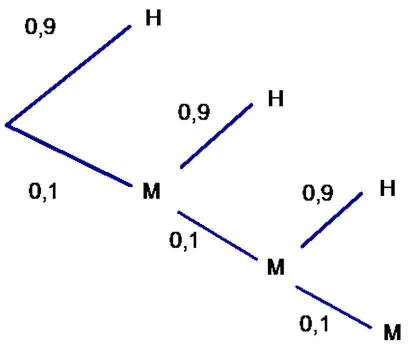
= 0,99

Third missile fired: P(hit) = 0,9 + MH + MMH

= 0,9 + (0,1 x 0,9) + (0,1 x 0,1 x 0,9)

= 0,999 Hence Lulu was correct

Alternate:



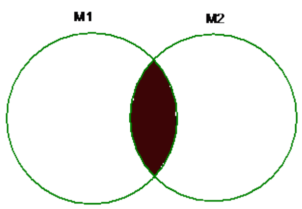
2 Missiles fired:

P(a hit) = 1 – P(no hit)
= 1 - P(MM)
= 1 - (0,1 x 0,1)
= 0,99

3 missiles fired:
P(a hit) = 1 – P(no hit)
= 1 - P(MMM)
= 1 - (0,1 x 0,1 x 0,1)
= 0,999

Hence Lulu was correct.

Alternate:



$P(M_1 \cup M_2) = P(M_1) + P(M_2) - P(M_1 \cap M_2)$
$= 0,9 + 0,9 - (0,9 \times 0,9)$
$= 0,99$

Similarly, if 3 missiles are fired:
$P(M_1 \cup M_2 \cup M_3) = 0,999$

Hence Lulu was correct

QUESTION 13

$y = -\frac{3}{2}x + 3$

Area ΔOMN $= \frac{1}{2}b.h$

Area ΔOMN $= \frac{1}{2}x\left(-\frac{3}{2}x + 3\right)$

Area ΔOMN $= -\frac{3}{4}x^2 + \frac{3}{2}x$

For max. value x_1 : $\frac{dA}{dx} = 0$

$0 = -\frac{3}{2}x_1 + \frac{3}{2}$

$\therefore x_1 = 1$

$f(x) = rx^2 + bx + c$ where $r = -\frac{3}{4}$

From: $f'(x) = -\frac{3}{2}x + 3$ …. By inspection, $b = 3$

$f(x) = -\frac{3}{4}x^2 + 3x + c$

Stationary point $(x;5)$

X-Intercept if f '(x) represents x-coordinate of the Stationary Point

$\therefore$ Stationary Point (2;5)

Substitute $(2;5)$ in $f(x) = -\frac{3}{4}x^2 + 3x + c$

$5 = -\frac{3}{4}(2)^2 + 3(2) + c$

$c = 2$

For value of x_2 that give max. distance (S) between f and f':

$S = -\frac{3}{4}x^2 + 3x + 2 - \left(-\frac{3}{2}x + 3\right)$

$S = -\frac{3}{4}x^2 + \frac{9}{2}x - 1$

$\frac{dS}{dx} = 0$

$-\frac{3}{2}x_2 + \frac{9}{2} = 0$

$x_2 = 3$

They differ.

Alternate:

$y = -\frac{3}{2}x + 3$

Area ΔOMN $= \frac{1}{2}b.h$

Area ΔOMN $= \frac{1}{2}x\left(-\frac{3}{2}x + 3\right)$

Area ΔOMN $= -\frac{3}{4}x^2 + \frac{3}{2}x$

For max. value x_1 : $\frac{dA}{dx} = 0$

$0 = -\frac{3}{2}x_1 + \frac{3}{2}$

$\therefore x_1 = 1$

$f(x) = rx^2 + bx + c$ where $r = -\frac{3}{4}$

From: $f'(x) = -\frac{3}{2}x + 3$ …. By inspection, $b = 3$

$f(x) = -\frac{3}{4}x^2 + 3x + c$

Stationary point $(x;5)$

X-Intercept if f ’(x) represents x-coordinate of the Stationary Point

$\therefore$ Stationary Point (2;5)

Substitute $(2;5)$ in $f(x) = -\frac{3}{4}x^2 + 3x + c$

$5 = -\frac{3}{4}(2)^2 + 3(2) + c$

$c = 2$

For value of x_2 that give max. distance (S) between f and f':

$S(x) = -\frac{3}{4}x^2 + 3x + 2 - \left(-\frac{3}{2}x + 3\right)$

$S(x) = -\frac{3}{4}x^2 + \frac{9}{2}x - 1$

$S(1) = 2,75$ and $S(2) = 5$,

Hence maximum distance is not at $x = 1$

Total: 150 marks