



GRAAD 12-EKSAMEN  
NOVEMBER 2018

**GEVORDERDEPROGRAM-WISKUNDE: VRAESTEL II**

**NASIENRIGLYNE**

Tyd: 1 uur

100 punte

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**Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.**

**Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.**

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**MODULE 2 STATISTIEK**

**VRAAG 1**

1.1 (a) Tweeterm

$$\begin{aligned} \text{Nul of een } & \binom{12}{0}(0,057)^0(0,943)^{12} + \binom{12}{1}(0,057)(0,943)^{11} \\ & = 0,8531 \end{aligned}$$

(b) Die oorblywende 16 het nie AGS nie

$$(0,943)^{16} = 0,3910$$

1.2 (a) Sonder vervanging hipergeometries

(b) 20

(c) 7

(d) 8

(e) 2

(f)  $7 - k$      $8 - (7 - k) = k + 1$

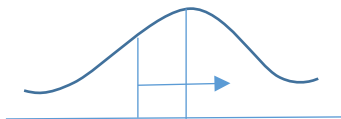
**VRAAG 2**

2.1 (a) Laat  $X$  die stogastiese veranderlike "gewig van babas" wees.

$$P(X > 2,8)$$

$$P\left(z > \frac{2,8 - 3,2}{0,85}\right)$$

$$P(z > -0,4706)$$

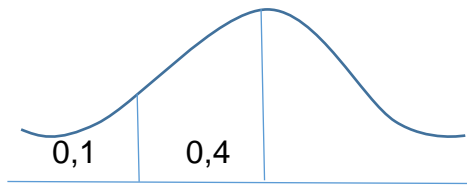


$$0,05 + 0,1808$$

$$= 0,6808$$

Ongeveer 953 120 babas

(b)



$$z = -1,28$$

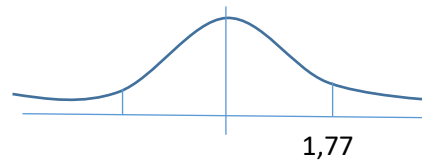
$$-1,28 = \frac{(X - 3,2)}{0,85}$$

$$= 2,112 \text{ kg}$$

2.2 (a) 61 kg

(b)  $61 + z \times \frac{9}{8} = 63$

$$z = \frac{16}{9} = 1,77$$



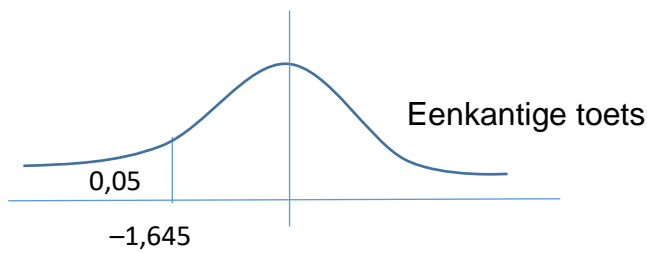
$0,461 \times 2$   
92% vertroue

**VRAAG 3**

$$H_0 : \mu_x = \mu_y$$

$$H_1 : \mu_x > \mu_y$$

Toetsstatistiek  $z = \frac{7,2 - 8,1}{\sqrt{\frac{(2,85)^2}{35} + \frac{4}{38}}} = -1,54$



Nie genoeg bewys om die nulhipotese ten gunste van die bewering by die 5%-betekenispeil te verwerp nie.

**VRAAG 4**

$$4.1 \quad \bar{y} = \frac{\sum y}{n} = 159 \frac{1}{6} = \frac{1910}{n}$$

$$n = 12$$

$$4.2 \quad b = \frac{12 \times 26270 - 161 \times 1910}{12 \times 2293 - (161)^2} = 4,8464$$

$$\bar{y} = a + b\bar{x}$$

$$\frac{955}{6} = a + 4,8464 \left( \frac{161}{12} \right) \therefore a = 94,1441$$

$$y = 94,1441 + 4,8464x$$

4.3 Sterk, positiewe korrelasie

4.4 Nee – te ver buite die beeldruimte (ekstrapolasie)

**VRAAG 5**

$$5.1 \quad \int_{30}^{60} a(x-30)^2 dx = 1$$

$$\left[ \frac{a}{3}(x-30)^3 \right]_{30}^{60} = 1$$

$$\frac{a}{3}(30)^3 = 1 \checkmark \checkmark$$

$$a = \frac{3}{(30)^3} = \frac{1}{9\,000}$$

$$5.2 \quad \left[ \frac{1}{27000}(x-30)^3 \right]_{30}^m = \frac{1}{2}$$

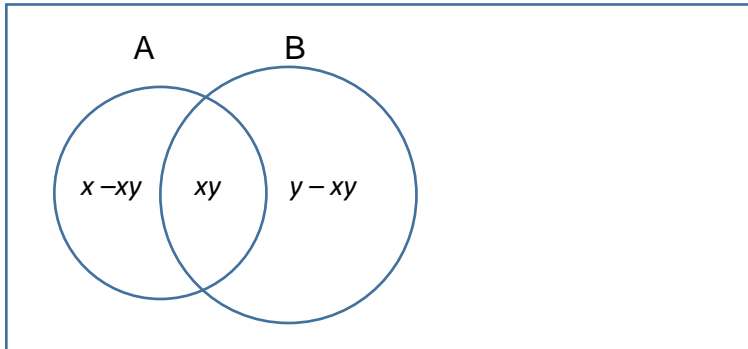
$$\frac{1}{27000}(m-30)^3 = \frac{1}{2}$$

$$(m-30)^3 = 1\,3500 \text{ m}$$

$$m = 54 \text{ minute}$$

**VRAAG 6**

6.1  $P(A) = x$   $P(B) = y$   $P(B') = 1 - y$   
 $P(A \cap B) = xy$



$P(A) \times P(B') = x(1 - y)$   
 $P(A \cap B') = x - xy = x(1 - y)$   
 Gebeurtenisse A en B' is onafhanklik.

6.2  $\binom{16}{5} - \binom{9}{0} \binom{7}{5} = 4\,347$

6.3  $\binom{11}{2} \binom{9}{4} \binom{5}{5} + \binom{11}{1} \binom{10}{4} \binom{6}{6} + \binom{11}{2} \binom{9}{3} \binom{6}{6} = 13\,860$

**Totaal vir Module 2: 100 punte**

**MODULE 3 FINANSIES EN MODELLERING**

**VRAAG 1**

1.1 (a)  $5\,640 \times \frac{1}{1,15} = \mathbf{4\,904,35\ B}$

(b)  $\frac{1,15 - 1,14}{1,14} \times 100 = 0,00877 \dots = \mathbf{0,88\%}$

1.2  $3x = x(1 + i)^{24} \qquad \therefore i = 0,0468 \text{ per maand}$

$2x = x(1 + 0,0468)^n \qquad \therefore n = 15,1423 \approx \mathbf{15 \text{ maande}}$

1.3 A 3      B 2      C 1      D 5

**VRAAG 2**

2.1  $500\,000 \left( \frac{0,088}{12} \right) = 3\,666,67 \qquad \text{rente} > \text{betalings}$

2.2  $500\,000 = \frac{x \left[ 1 - \left( 1 + \frac{0,088}{12} \right)^{-96} \right]}{\frac{0,088}{12}} \qquad \mathbf{x = 7\,273,33}$

2.3  $500\,000 \left( 1 + \frac{0,088}{12} \right)^{95} - \frac{7\,300 \left[ \left( 1 + \frac{0,088}{12} \right)^{95} - 1 \right]}{\frac{0,088}{12}} = 3\,576,4053$

$3\,576,4053 \left( 1 + \frac{0,088}{12} \right) = \mathbf{3\,602,63}$

**OF**

$$500\,000 \left( 1 + \frac{0,088}{12} \right)^{96} - \frac{7\,300 \left( 1 + \frac{0,088}{12} \right) \left[ \left( 1 + \frac{0,088}{12} \right)^{95} - 1 \right]}{\frac{0,088}{12}}$$

$= 1\,008\,318,445 - 1\,004\,715,812 = \mathbf{3\,602,63}$

**OF**

$$500\,000 - \frac{7\,300 \left[ 1 - \left( 1 + \frac{0,08}{12} \right)^{-95} \right] + y \left( 1 + \frac{0,08}{12} \right)^{-96}}{\frac{0,08}{12}}$$

$$\therefore y = \mathbf{3\,602,63}$$

**VRAAG 3**

$$x \cdot \left( 1 + \frac{0,08}{12} \right)^{72} \cdot \frac{2}{3} \cdot \left( 1 + \frac{0,08}{12} \right)^{24} + x \cdot \left( 1 + \frac{0,08}{12} \right)^{72} \cdot \frac{1}{3} \cdot \left( 1 + \frac{0,1}{4} \right)^8$$

$$= 20\,702,50$$

$$1,9169x = 20\,702,50 \quad \mathbf{X = 10\,800}$$

**VRAAG 4**

4.1 Logistiese Model, aanwesigheid van dra vermoë B

$$4.2 \quad V = \frac{1}{2} (50) = \mathbf{25 \text{ R}}$$

4.3 Die model het regressievergelyking  $\frac{\Delta P}{P} = -0,0025P + r$ .  
 $r = -Km = -50 \cdot (-0,0025) = \mathbf{0,125}$

$$4.4 \quad T_{n+1} = T_n + 0,13 \cdot T_n(1 - T_n/50), \quad T_0 = 10$$

$\mathbf{t = 11}$

**VRAAG 5**

- 5.1 (a) Getal arende wat per jaar gebore word  
 (b) Doeltreffendheidskoers ✓ waarteen arende prooi in nageslag verander

(c)  $f \cdot b \cdot D_n \cdot E_n = 15$   
 $f(6\ 000) = 15$   **$f = 0,0025$**

5.2  $a = 0,5 \times 1,5 \times 3 \times 0,67$   **$a = 1,51$**

5.3  $6\ 000 = b \cdot (12\ 000)(30)$   **$b = 0,016\ 667$**   
 vir dassie-ewewig,  $E_{n+1} = E_n$   
 $0,1 = 0,003 \times 0,016\ 667 \times D$   **$D = 1\ 999,96 \approx 2\ 000$**

**OF**

$6\ 000 = b \cdot (12\ 000)(30)$   **$b = 0,016\ 667$**   
 $D = \frac{c}{fb} = \frac{0,1}{0,003 \times 0,016\ 667}$   **$D = 1\ 999,96 \approx 2\ 000$**

**VRAAG 6**

6.1 (a)  $T_4 = 75,77$      $T_5 = 84,55$      $T_6 = 91,122$

(b)  $64\sqrt{3} = 110,8$  vierkante eenhede

6.2  $195 = p \cdot 114 + q \cdot 60$     en     $114 = p \cdot 60 + q \cdot 24$   
 $p = 2,5$     en     $q = -1,5$   
 $T_n = 5/2 \cdot T_{n-1} - 3/2 \cdot T_{n-2}$      **$T_1 = 24, T_2 = 60$**

**Totaal vir Module 3: 100 punte**



**MODULE 4                    Matrikse en Grafiekteorie**

**Vraag 1**

$$1.1 \quad PQ = \begin{pmatrix} 3 & 6 & 2 & -2 \\ 0 & -1 & 4 & 6 \end{pmatrix} \cdot \begin{pmatrix} 3 & 6 \\ -2 & -1 \\ 0 & 5 \\ -7 & 0 \end{pmatrix} = \begin{pmatrix} 11 & 22 \\ -40 & 21 \end{pmatrix}$$

$$1.2 \quad 3x + 2y = 11 \qquad x - 2z = 0 \qquad 6y + 4z = 5$$

3	2	0	11	
1	0	-2	0	
0	6	4	5	
3	2	0	11	
0	2	6	11	R1 - 3.R2
0	6	4	5	
3	2	0	11	
0	2	6	11	
0	0	14	28	3.R2 - R3

$$z = 2, \quad y = -\frac{1}{2}, \quad x = 4$$

- 1.3 (a) 3  
 (b) 0  
 (c)  $t$

**VRAAG 2**

2.1   refleksie in  $y = x$

2.2    $k = 3$

2.3    $C = \frac{1}{4} R$  en  $R = S$  dus is faktor  $\frac{1}{4}$

2.4    $\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -0,5 & 0 \\ 0 & -0,5 \end{pmatrix} = \begin{pmatrix} 1,5 & 0 \\ 0 & -0,5 \end{pmatrix}$

OF

$$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0,5 & 0 \\ 0 & 0,5 \end{pmatrix} = \begin{pmatrix} 1,5 & 0 \\ 0 & -0,5 \end{pmatrix}$$

2.5    $\begin{pmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{pmatrix} \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 4,025 \\ 3,578 \end{pmatrix}$

$$\begin{aligned} 5\cos A + 2.\sin A &= 4,025 && \text{en} \\ \cos A &= 0,4472 && \text{en} \\ \mathbf{A} &= \mathbf{63,44^\circ} \end{aligned}$$

$$\begin{aligned} -2.\cos A + 5\sin A &= 3,578 \\ \sin A &= 0,8944 \\ \mathbf{A} &= \mathbf{63,44^\circ} \end{aligned}$$

**VRAAG 3**

3.1    $\det = 25$

3.2    $\begin{pmatrix} 25 & 0 & 0 & -1 & -4 & -10 \\ 0 & -10 & 0 & -2 & -8 & -10 \\ 0 & 0 & 25 & 4 & -9 & 10 \end{pmatrix}$

3.3    $\begin{pmatrix} 25 & 0 & 0 & -1 & -4 & 10 \\ 0 & 25 & 0 & 5 & 20 & -25 \\ 0 & 0 & 25 & -4 & 9 & -10 \end{pmatrix}$

$$\text{Inverse} = \frac{1}{25} \begin{pmatrix} -1 & -4 & 10 \\ 5 & 20 & -25 \\ -4 & 9 & -10 \end{pmatrix}$$

**VRAAG 4**

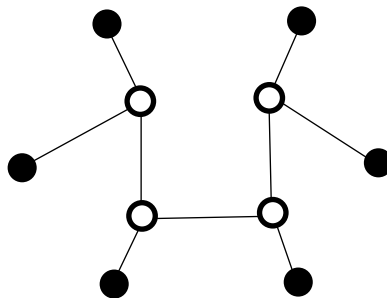
- 4.1 Een nodus het 'n onewe graad.
- 4.2 Ja; daar is een paar onewe nodusse.
- 4.3 8 skakels
- 4.4  $19 \times 2 = 4 \times 6 + 2 \times 4 + 1 \times 1 + e$   
 **$e = 5$**

**VRAAG 5**

- 5.1 TR 3 TV 3 TS 4 SU 3  
RQ 5 RW 5 QP 6 lengte = 29
- 5.2 RT 3 TV 3 VW 7  
WRQ 10 QP 6 PS 7  
SU 3 UTR 9 **U/B = 48**
- 5.3 37 is die grootste ondergrens en 41 is die kleinste bogrens
- 5.4 R Q P U S T V W R = 41

**VRAAG 6**

- 6.1 3
- 6.2  $e = 2n - 3$
- 6.3 4 Steiner-nodusse  
9 skakels  
Samehangendheid



**Totaal vir Module 4: 100 punte**