GRAAD 12-EKSAMEN
NOVEMBER 2018

GEVORDERDEPROGRAM-WISKUNDE: VRAESTEL I
MODULE 1: CALCULUS EN ALGEBRA

NASIENRIGLYNE

Tyd: 2 uur 200 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

VRAAG 1

1.1 (a)

$$\left|x^2+x\right| = -2x-2$$
$$\therefore x^2+x=-2x-2 \text{ of } -x^2-x=-2x-2$$
$$\therefore x^2+3x+2=0 \text{ of } x^2-x-2=0$$
$$\therefore x=-1 \text{ of } -2 \text{ of } x=-1 \text{ of } 2$$
$$\textit{Maar kontrolering toon } x=-1 \text{ of } -2$$

(b)

$$\ln x^3 + 2\ln x^2 = 7$$
$$\therefore \ln x^3 + \ln x^4 = 7$$
$$\therefore \ln x^7 = 7 \ (\textit{kan van hier af reguit na die antwoord gaan})$$
$$\therefore 7\ln x = 7$$
$$\therefore \ln x = 1$$
$$\therefore x = e$$

1.2 (a)

$$y = y_0 e^{-kt}$$
$$\therefore e^{-kt} = \frac{y}{y_0}$$
$$\therefore -kt = \ln\frac{y}{y_0}$$
$$\therefore k = \frac{\ln\frac{y}{y_0}}{-t}$$

(b)

$$k = \frac{\ln\frac{0,5y_0}{y_0}}{-5700} \checkmark\text{m} \approx 1{,}216\times10^{-4}$$

(c)

$$0,9y_0 = y_0 e^{-kt}$$
$$\therefore -kt = \ln 0,9$$
$$\therefore t = \frac{\ln 0,9}{-k}$$
$$\therefore t \approx 866 \ \textit{jaar}$$

VRAAG 2

2.1 *Indien* $3+2i$ *'n wortel s, dan is* $3-2i$ *ook*

Dus is ons vergelyking :

$$(x+3)\big(x-(3+2i)\big)\big(x-(3-2i)\big)=0$$

$$\therefore (x+3)\big((x-3)-2i\big)\big((x-3)+2i\big)=0$$

$$\therefore (x+3)\Big((x-3)^2-4i^2\Big)=0$$

$$\therefore (x+3)\left(x^2-6x+13\right)=0$$

$$\therefore x^3-3x^2-5x+39=0$$

2.2 'n Derdegraadsvergelyking sal drie wortels hê. Komplekse wortels van polinome met reële koëffisiënte kom voor in toegevoegde pare, dus moet daar minstens een reële wortel wees.

2.3 $$\frac{a+bi}{-b+ai}\times\frac{-b-ai}{-b-ai}$$

$$=\frac{-ab-a^2i-b^2i-abi^2}{b^2-a^2i^2}$$

$$=\frac{ab-ab-i\left(a^2+b^2\right)}{b^2+a^2}$$

$$=\frac{-i\left(a^2+b^2\right)}{b^2+a^2}$$

$$=-i$$

VRAAG 3

Indien $n = 1$, *het ons* $2^3 - 3 = 5$ *wat duidelik deelbaar is deur* 5.

Neem aan waar vir $n = k$, *naamlik dat*

$2^{3k} - 3^k = 5p$ waar $p \in \mathbb{N}$ $\quad (*)$

Indien $n = k + 1$, *het ons*:

$2^{3(k+1)} - 3^{k+1}$

$= 2^{3k+3} - 3^{k+1}$

$= 2^{3k} \times 2^3 - 3 \times 3^k$

$= 8 \times 2^{3k} - 3 \times 3^k$

Uit $(*)$ *het ons* $2^{3k} = 5p + 3^k$, *dus indien* $n = k + 1$ *het ons*

$= 8(5p) + 8(3^k) - 3 \times 3^k$

$= 5(8p) + 5(3^k)$

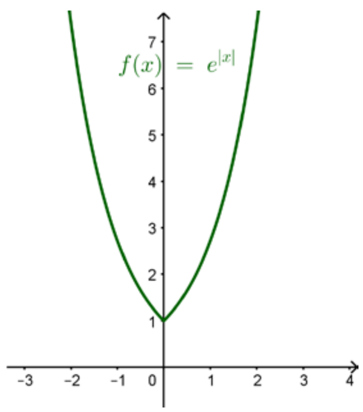
$= 5(8p + 3^k)$

wat duidelik deelbaar is deur 5

Deur die beginsel van volledige induksie het ons dus die resultaat bewys vir $n \in \mathbb{N}$

VRAAG 4

4.1 (a)



(b) $x = 0$

4.2 *Indien* f *differensieerbaar is by* $x = 2$
moet dit kontinu wees by $x = 2$
dus $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$
dus $2a - b - 1 = 4b - 2a + 5$ *of* $4a - 5b = 6 \ (1)$
maar ook $\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^+} f'(x)$
$a = 4b - a$ *of* $a = 2b \ (2)$
Los (1) *en* (2) *gelyktydig op*:
$8b - 5b = 6$ *dus* $b = 2$ *en* $a = 4$

VRAAG 5

5.1 *segment* $=$ *sektor* $- \Delta$
$\therefore 308 = \frac{1}{2}(18^2)\theta - \frac{1}{2}(18^2)\sin\theta$
$\therefore 162\theta - 162\sin\theta - 308 = 0$

5.2 $f(\theta) = 162\theta - 162\sin\theta - 308$
$\therefore \theta_{n+1} = \theta_n - \dfrac{162\theta - 162\sin\theta - 308}{162 - 162\cos\theta}$
$\theta = 2{,}49984$

VRAAG 6

6.1 $f(0) = \frac{1}{2}$, *dus* y *- afsnit* $\left(0;\frac{1}{2}\right)$
$\dfrac{2x^2 - 3x - 2}{x - 4} = 0$
$\therefore 2x^2 - 3x - 2 = 0$
$\therefore (2x + 1)(x - 2) = 0$
$\therefore x$ *- afsnitte* $\left(-\frac{1}{2};0\right)$ *en* $(2;0)$

6.2 *Vertikale asimptoot* : $x = 4$
$2x^2 - 3x - 2 = (x - 4)(2x + 5) + R$
Dus is skuins asimptoot $y = 2x + 5$

6.3 $f(x) = \dfrac{2x^2 - 3x - 2}{x - 4}$

$\therefore f'(x) = \dfrac{(4x-3)(x-4) - 1(2x^2 - 3x - 2)}{(x-4)^2} = 0$

$\therefore 4x^2 - 19x + 12 - 2x^2 + 3x + 2 = 0$

$\therefore 2x^2 - 16x + 14 = 0$

$\therefore x^2 - 8x + 7 = 0$

$\therefore (x-1)(x-7) = 0$

$\therefore x = 1 \quad of \quad 7$

$\therefore (1;1)$ *en* $(7;25)$ *is stasionêre punte*

6.4 $f''(1) < 0$ *dus is* $(1;1)$ *'n lokale maksimum*

$f''(7) > 0$ *dus is* $(7;25)$ *'n lokale minimum*

VRAAG 7

7.1 $x^2 + xy + y^2 = 1$

$\therefore 2x + y + x\dfrac{dy}{dx} + 2y\dfrac{dy}{dx} = 0$

$\therefore \dfrac{dy}{dx}(x + 2y) = -2x - y$

$\therefore \dfrac{dy}{dx} = \dfrac{-2x - y}{x + 2y}$

7.2 *By A*: $y = 0 \quad \therefore \quad x = 1$

Dus by A: $\dfrac{dy}{dx} = \dfrac{-2}{1} = -2$

$\therefore y = -2(x-1)$

$\therefore y = -2x + 2$

VRAAG 8

8.1 $\cos\theta = \dfrac{FC}{CD}$

$\therefore FC = 0,4\cos\theta$

$\therefore A = \Delta CDF + \Delta BEG + BCFG$

$\therefore A = 2\left(\dfrac{1}{2}\times 0,4\times 0,4\cos\theta\sin\theta\right) + 0,4\times 0,4\cos\theta \quad (\Delta CDF = \Delta BEG)$

$\therefore A = 0,16\sin\theta\cos\theta + 0,16\cos\theta$

$\therefore A = 0,08\sin 2\theta + 0,16\cos\theta$

$\therefore V = 20(0,08\sin 2\theta + 0,16\cos\theta)$

$\therefore V = 1,6\sin 2\theta + 3,2\cos\theta$

8.2 $V = 1,6\sin 2\theta + 3,2\cos\theta$

$\therefore \dfrac{dV}{d\theta} = 3,2\cos 2\theta - 3,2\sin\theta = 0$

$\therefore \cos 2\theta = \sin\theta$

$\therefore \cos 2\theta = \cos\left(\dfrac{\pi}{2} - \theta\right)$

$\therefore 2\theta = \dfrac{\pi}{2} - \theta$

$\therefore 3\theta = \dfrac{\pi}{2}$

$\therefore \theta = \dfrac{\pi}{6}$

VRAAG 9

9.1 (a) $\sin^3\theta = \sin\theta\times\sin^2\theta$

$= \sin\theta(1-\cos^2\theta)$

$= \sin\theta - \sin\theta\cos^2\theta$ *soos gevra*

(b) $\int \sin^3\theta\, d\theta = \int \sin\theta\, d\theta - \int \sin\theta\cos^2\theta\, d\theta$

$= -\cos\theta + \dfrac{\cos^3\theta}{3} + c$

9.2 **METODE 1**

$$\int \frac{x}{\sqrt{2+x}}\,dx$$

Laat $u = 2 + x$ *dan* $x = u - 2$ *en* $du = dx$

$$\therefore \int \frac{u-2}{u^{\frac{1}{2}}}\,du$$

$$= \int u^{\frac{1}{2}} - 2u^{-\frac{1}{2}}\,du$$

$$= \frac{2}{3}u^{\frac{3}{2}} - 4u^{\frac{1}{2}} + c$$

$$= \frac{2}{3}(2+x)^{\frac{3}{2}} - 4(2+x)^{\frac{1}{2}} + c$$

ALTERNATIEF 1

$$\int \frac{x}{\sqrt{2+x}}\,dx$$

$$= \int x(2+x)^{-\frac{1}{2}}\,dx$$

Parsieel $f = x$ *en* $g' = (2+x)^{-\frac{1}{2}}$

Dus $f' = 1$ *en* $g = 2(2+x)^{\frac{1}{2}}$

$$= 2x(2+x)^{\frac{1}{2}} - \int 2(2+x)^{\frac{1}{2}}\,dx$$

$$= 2x(2+x)^{\frac{1}{2}} - \frac{4(2+x)^{\frac{3}{2}}}{3} + c$$

VRAAG 10

10.1 $Oppervlakte = \frac{10}{3} + \frac{3}{2(4)} + \frac{1}{6(4^2)}$

$= \frac{119}{32}$

10.2 Dit sal 'n oorbenadering wees. Namate *n* groter word, word die antwoord kleiner.

10.3 $Oppervlakte = \lim_{n\to\infty}\left(\frac{10}{3} + \frac{3}{2n} + \frac{1}{6n^2}\right)$ m vir *n* wat na oneindigheid neig

$= \frac{10}{3}$ $eenhede^2$

10.4 $\frac{10}{3}$ $eenhede^2$ aangesien dit bloot 'n refleksie van die gearseerde gebied in die *y*-as is.

VRAAG 11

$$Oppervlakte = \int_1^7 g(x) - f(x)\,dx$$

$$= \int_1^7 f(x) + kx + 1 - f(x)\,dx$$

$$= \int_1^7 kx + 1\,dx$$

$$= \left[\frac{kx^2}{2} + x\right]_1^7$$

$$dus \quad \frac{49k}{2} + 7 - \frac{k}{2} - 1 = 54$$

$$dus \quad 49k + 14 - k - 2 = 108$$

$$dus \quad 48k = 96$$

$$dus \quad k = 2$$

VRAAG 12

$$vol = \pi\int_a^b \left[f(x)\right]^2 dx$$

$$\therefore 175 = \pi\int_a^b -x^2 + 6x + 4\,dx$$

$$\therefore 175 = \pi\left[-\frac{x^3}{3} + 3x^2 + 4x\right]_0^h$$

$$\therefore 175 = \pi\left(-\frac{h^3}{3} + 3h^2 + 4h\right)$$

$$\therefore -h^3 + 9h^2 + 12h - \frac{525}{\pi} = 0$$

$$\therefore x = 5{,}28 \text{ cm}$$

Totaal: 200 punte