

**GEVORDERDEPROGRAM-WISKUNDE: VRAESTEL II**

**NASIENRIGLYNE**

Tyd: 1 uur

100 punte

---

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en sub-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

---

**MODULE 2            STATISTIEK**

**VRAAG 1**

1.1 
$$\frac{\binom{5}{3}\binom{7}{2}}{\binom{12}{5}} = 0.2652$$

1.2 
$$P(X = 7) = \binom{10}{7}(0.7)^7 (0.3)^3 = 0.2668$$

1.3 (a) 
$$\frac{5!}{2!2!} + 2 \times \frac{5!}{2!3!} = 50$$

(b) 
$$5 \times 2 + 2 = 12 \{(\text{CHLHL}) \times 5 + (\text{HLCLH}) \times 2 = 12\}$$

**VRAAG 2**

2.1 (a) 
$$0,3 \times (0,7) + (0,3)(0,7)^2 + (0,3)(0,7)^3 + (0,3)(0,7)^4 + C = 1$$
  

$$C = 0,4681$$

(b) 
$$P(X > 3) = P(X = 4) + P(X = 5)$$
  

$$= 0,3(0,7)^4 + 0,4681$$
  

$$= 0,5401$$

2.2 (a) A

(b) (i) 
$$\frac{n}{500} = \frac{0,2278 + 0,2922}{2}$$
  

$$\therefore n = 130$$

(ii) 
$$0,26 + Z\sqrt{\frac{(0,26)(0,74)}{500}} = 0,2922$$
  

$$Z = 1,64$$
  

$$\therefore \alpha = 90$$

**VRAAG 3**

3.1  $X \sim N(9, 0,1^2)$

$$\begin{aligned} P(X > 8,9) &= P\left(Z > \frac{8,9-9}{0,1}\right) \\ &= P(Z > -1) \\ &= 0,5 + 0,3413 \\ &= 0,8413 \end{aligned}$$

3.2  $X \sim B(6, 0,8413)$

$$\begin{aligned} P(X \geq 2) &= 1 - \left[ \binom{6}{0}(0,8413)^0(0,1587)^6 + \binom{6}{1}(0,8413)^1(0,1587)^5 \right] \\ &= 0,9995 \end{aligned}$$

3.3  $P(X < a) = 0,04$

$$-1,75 = \frac{a-9}{0,1}$$

$$\therefore a = 8,825 \text{ cm}$$

**VRAAG 4**

4.1 (a)  $1 + m^2 + (m + 1)^2 + 4^2 + 5^2 = 55$

$$2m^2 + 2m - 12 = 0$$

$$m^2 + m - 6 = 0$$

$$(m + 3)(m - 2) = 0$$

$$m \neq -3 \text{ of } m = 2$$

$$\frac{5+t-1+4+3+t}{5} = 3$$

$$2t + 11 = 15$$

$$2t = 4$$

$$t = 2$$

(b)  $r = -0,4$

(c) (i)  $y = 4,2 - 0,4x$

(ii)  $y = 4,2 - 0,4(6)$

$$y = 1,8$$

Dit is 'n onbetroubare beraming want die korrelasie is swak.

4.2  $H_0 : \mu = 49,5$

$H_1 : \mu < 49,5$

Verwerpingsgebied:

Verwerp  $H_0$  indien  $Z < -1,48$

Toetsstatistiek:

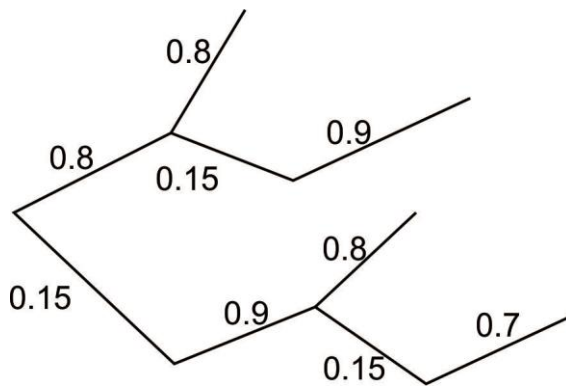
$$Z = \frac{48 - 49,5}{\frac{4,8}{\sqrt{40}}} = -1,98$$

Gevolgtrekking: aangesien  $Z < -1,48$ , verwerp ons  $H_0$  by die 7%-betekenispeil en stel voldoende bewys voor om die bewering te ondersteun dat Basi se werksure per week aansienlik afgeneem het.

**VRAAG 5**

5.1  $0,8 + 0,15(0,9) = 0,935$

5.2



$$(0.8)(0.8) + (0.8)(0.15)(0.9) + (0.15)(0.9)(0.8) + (0.15)(0.9)(0.15)(0.7) = 0.8702$$

**Totaal vir Module 2: 100 punte**

**MODULE 3 FINANSIES EN MODELLERING**

**VRAAG 1**

- 1.1 R1 200 000
- 1.2 50
- 1.3 ± R1 100 000 metode: 2,3 – 1,2 = 1,1 miljoen
- 1.4 ± R900 000
- 1.5 (a) Reguitlyngradiënt nie beïnvloed nie  
 (b) Rente is verhoog  
**OF**  
 Onttrekking uit lening

**VRAAG 2**

$$2.1 \quad P \left( 1 + \frac{0,0568}{12} \right)^2 = \frac{5154,26 \left[ 1 - \left( 1 + \frac{0,0568}{12} \right)^{-34} \right]}{\frac{0,0568}{12}}$$

**P = R160 000**

$$2.2 \quad \text{Uitstaande saldo} = \frac{5154,26 \left[ 1 - \left( 1 + \frac{0,0568}{12} \right)^{-12} \right]}{\frac{0,0568}{12}}$$

**Uitstaande saldo = R59 989,47**

- 2.3 Jude het betaal:  $5\,154,26 \times 12 = 61\,851,12$
- Saldo het afgeneem:  $116\,674,09 - 59\,989,47 = 56\,684,62$
- Rente betaal:  $61\,851,12 - 56\,684,62 = \mathbf{R5\,166,50}$

**VRAAG 3**

3.1  $2\,000\,000 = A(1 + 0,068)^8$   **$A = 1\,181\,571,41$**

3.2  $1\,181\,571,41 = 3\,400\,000(1 - i)^8$   **$i = 12,38\%$**

3.3  $\left(1 + \frac{0,0764}{12}\right)^{12} = \left(1 + \frac{i}{2}\right)^2$   **$i = 7,7626\%$**

$$5\,500\,000 + 300\,000 \left(1 + \frac{0,077626}{2}\right)^6 = 5\,877\,003,11$$

$$x \frac{\left[\left(1 + \frac{0,077626}{2}\right)^9 - 1\right] \left(1 + \frac{0,077626}{2}\right)^4}{\frac{0,07626}{2}} = 12,264x$$

$5\,877\,003,611 = 12,264x$   **$x = 479\,200,70$**

**VRAAG 4**

4.1  $\frac{1\,396 - 1\,300}{1\,300} = 7,4\%$

4.2  $Q_{n+1} = 1,05 \cdot Q_n - 50, \quad Q_0 = 6\,500$

4.3  $A = 8\,020$                        $B = 8\,371$   
 $C = 8\,739$                        $D = 9\,126$   
 $E = 9\,532$                        $F = 9\,959$

4.4  $9\,126 / 4 = 2\,281 < 2\,301$  **gedurende 8<sup>ste</sup> jaar**

4.5  $\frac{2\,655 - 2\,472}{2\,472} = 7,4\%$  **konstante eksponensiële groei;/dus**  
**Malthusies**

**VRAAG 5**

5.1 (a) prooi  $\approx 526\ 000$  roofdier  $\approx 4\ 500$

(b)  $\pm 4\ 180 - 4190$

(c) B

5.2  $S_{n+1} = 4\ 000 + 760 - 0,2 \times 4\ 000 = \mathbf{3\ 960}$

5.3  $2/3 \times 3 \times 8 \times 0,05 = \mathbf{0,8}$

5.4  $533\ 300 = 500\ 000 + 0,8 (500\ 000) \left(1 - \frac{500\ 000}{K}\right) - \mathbf{0,4 (500\ 000)}$   
 $\mathbf{K = 1\ 200\ 000}$

**VRAAG 6**

6.1 (a)  $8^{\text{ste}}$

(b) A

6.2  $\frac{2a+3b}{a+2b} = \frac{a+2b+a+b}{a+2b} = 1 + \frac{a+b}{a+2b} = 1 + \frac{1}{\frac{a+b+b}{a+b}} = 1 + \frac{1}{1 + \frac{b}{a+b}}$   
 $= 1 + \frac{1}{1 + \frac{1}{\frac{a+b}{b}}} = 1 + \frac{1}{1 + \frac{1}{1 + \frac{a}{b}}}$

**OF**

$$\begin{aligned} \frac{2a+4b-b}{a+2b} &= 2 - \frac{b}{a+2b} = 2 - \frac{1}{\frac{a+2b}{b}} \\ &= 2 - \frac{1}{2 + \frac{a}{b}} \\ &= 2 - \frac{1}{2 + T_n} \end{aligned}$$

**Totaal vir Module 3: 100 punte**

**MODULE 4            MATRIKSE EN GRAFIEKTEORIE**

**VRAAG 1**

1.1 (a)  $1 \times 1$

(b)  $(k+3 \quad k+2 \quad 3) \begin{pmatrix} k \\ 3 \\ 1 \end{pmatrix} = (k^2 + 3k) + (3k+6) + 3$

**OF**

$(k \quad 1 \quad 1) \begin{pmatrix} k+3 \\ 3k+2 \\ 7 \end{pmatrix} = (k^2 + 3k) + (3k+2) + 7$

$k^2 + 6k + 9 = 0 \qquad \qquad \qquad \mathbf{k = -3}$

1.2 (a) 1

(b) ***pr***

(c) ***27p***

(d) ***pr/q*** (8)

**VRAAG 2**

2.1  $\begin{pmatrix} 1 & -3 & 6 & 4 \\ 0 & 3 & -1 & 1 \\ 0 & 2 & 2 & -18 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -3 & 6 & 4 \\ 0 & 3 & -1 & 1 \\ 0 & 8 & 0 & -16 \end{pmatrix}$

**$y = -2; \quad z = -7$**

$x - 3(-2) + 6(-7) = 4 \qquad \qquad \mathbf{x = 40}$

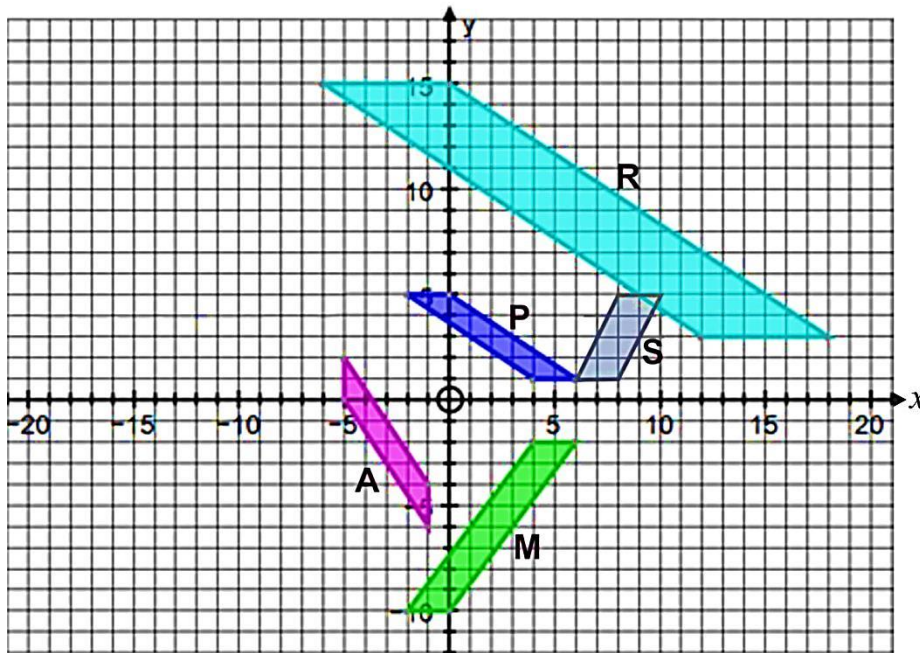
2.2 ***L:***  $\mathbf{0x + 0y + 0z \neq 1}$  (vergelyking is 'n strydigheid)

2.3 ***D:***  $\mathbf{0x + 0y + 0z = 0}$



**VRAAG 3**

3.1



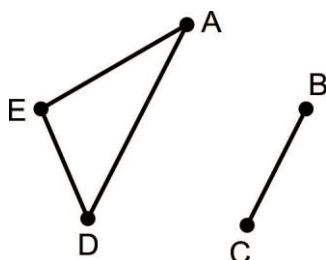
- (a) Refleksie; rigting  $y = -x$ ; koördinate
- (b) Vergroting; skaal  $k = 3$ ; koördinate
- (c) Strekking; invariante lyn  $y = 0$ ; skaal  $k = 2$ ; koördinate
- (d) Skuiwing, invariante lyn  $y = 0$ ; skaal  $k = 2$ , koördinate

3.2 
$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 6 \\ 1 \end{pmatrix} = \begin{pmatrix} -5,55 + 10 \\ 5,15 - 1 \end{pmatrix} = \begin{pmatrix} 4,45 \\ 4,15 \end{pmatrix}$$

$6\cos \Theta - \sin \Theta = 4,45$	EN	$\cos \Theta + 6\sin \Theta = 4,15$
$\cos \Theta = 0,83378378\dots$	OF	$\sin \Theta = 0,55270270\dots$
$\Theta = 33,5^\circ$		$\Theta = 33,6^\circ$

**VRAAG 4**

- 4.1 6 skakels
- 4.2 Nie simmetries nie
- 4.3 Alle nodusse het lus na hulself
- 4.4

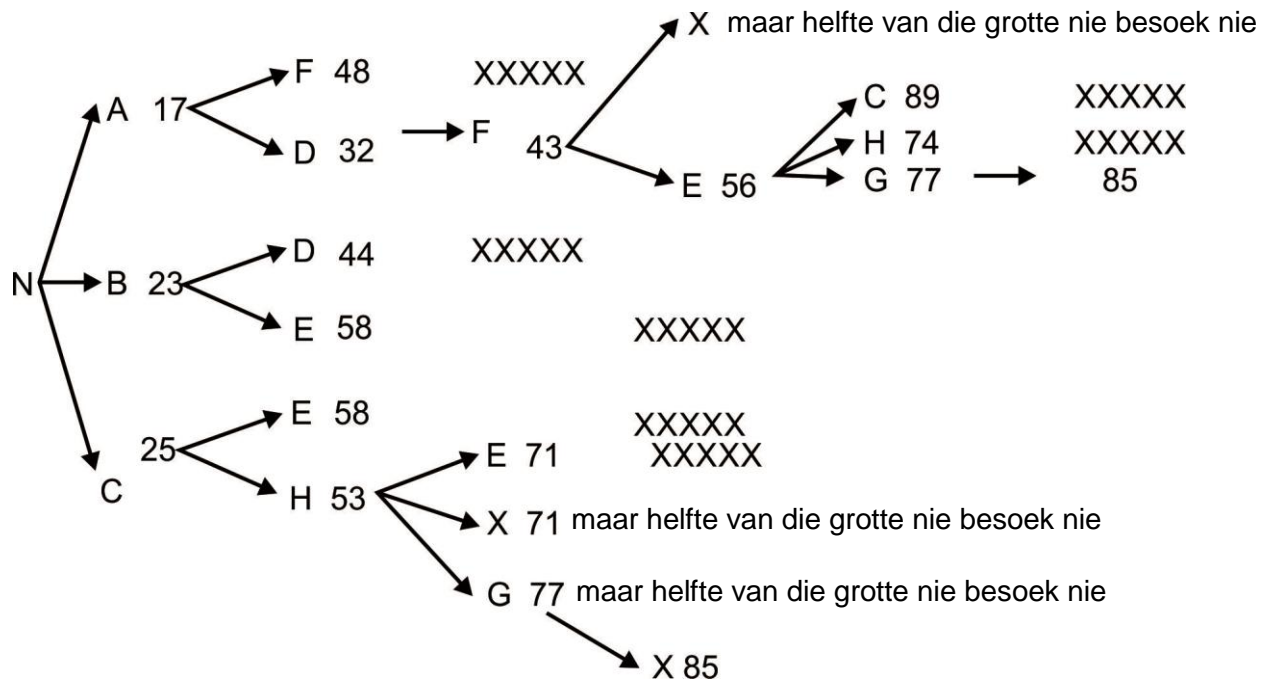


Vyf nodusse, vier skakels  
korrekte skakels, niesamehangend

**VRAAG 5**

- 5.1
- |    |    |                  |
|----|----|------------------|
| NA | 17 |                  |
| AD | 15 |                  |
| DF | 11 |                  |
| DB | 21 |                  |
| FX | 24 |                  |
| XG | 8  |                  |
| XH | 18 |                  |
| NC | 25 |                  |
| EF | 13 |                  |
| EH | 18 | <b>= 170 min</b> |

5.2



∴ **N A D F E G X = 85 min**

**OF**

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>G</b>	<b>H</b>	<b>X</b>	
<b>N</b>	<b>N<sub>17</sub></b>	<b>N<sub>23</sub></b>	<b>N<sub>25</sub></b>							
<b>A</b>	<b>N<sub>17</sub></b>	<b>N<sub>23</sub></b>	<b>N<sub>25</sub></b>	<b>A<sub>32</sub></b>	•	<b>A<sub>48</sub></b>				
<b>B</b>	<b>N<sub>17</sub></b>	<b>N<sub>23</sub></b>	<b>N<sub>25</sub></b>	<b>B<sub>44</sub></b>	<b>B<sub>58</sub></b>	<b>A<sub>48</sub></b>				
<b>C</b>	<b>N<sub>17</sub></b>	<b>N<sub>23</sub></b>	<b>N<sub>25</sub></b>	<b>A<sub>32</sub></b>	<b>C<sub>58</sub></b>	<b>A<sub>48</sub></b>	•	<b>C<sub>53</sub></b>	•	
<b>D</b>	<b>N<sub>17</sub></b>	<b>N<sub>23</sub></b>	<b>N<sub>25</sub></b>	<b>A<sub>32</sub></b>	<b>B/C<sub>58</sub></b>	<b>D<sub>43</sub></b>	•	<b>C<sub>53</sub></b>	•	
<b>F</b>	<b>N<sub>17</sub></b>	<b>N<sub>23</sub></b>	<b>N<sub>25</sub></b>	<b>A<sub>32</sub></b>	<b>F<sub>56</sub></b>	<b>D<sub>43</sub></b>	•	<b>C<sub>53</sub></b>	<b>F<sub>67</sub></b>	Maar helfte van grotte nie besoek nie
<b>H</b>	<b>N<sub>17</sub></b>	<b>N<sub>23</sub></b>	<b>N<sub>25</sub></b>	<b>A<sub>32</sub></b>	<b>H<sub>71</sub></b>	<b>D<sub>43</sub></b>	<b>H<sub>77</sub></b>	<b>C<sub>53</sub></b>	<b>H<sub>71</sub></b>	Maar helfte van grotte nie besoek nie
<b>E</b>	<b>N<sub>17</sub></b>	<b>N<sub>23</sub></b>	<b>N<sub>25</sub></b>	<b>A<sub>32</sub></b>	<b>F<sub>56</sub></b>	<b>D<sub>43</sub></b>	<b>E<sub>77</sub></b>	<b>C<sub>53</sub></b>	<b>H<sub>71</sub></b>	
<b>G</b>	<b>N<sub>17</sub></b>	<b>N<sub>23</sub></b>	<b>N<sub>25</sub></b>	<b>A<sub>32</sub></b>	<b>F<sub>56</sub></b>	<b>D<sub>43</sub></b>	<b>H/E<sub>77</sub></b>	<b>C<sub>53</sub></b>	<b>G<sub>85</sub></b>	
<b>G</b>	<b>N<sub>17</sub></b>	<b>N<sub>23</sub></b>	<b>N<sub>25</sub></b>	<b>A<sub>32</sub></b>	<b>F<sub>56</sub></b>	<b>D<sub>43</sub></b>	<b>H<sub>77</sub></b>	<b>C<sub>53</sub></b>	<b>G<sub>85</sub></b>	Maar helfte van grotte nie besoek nie
	∴ <b>N</b>	<b>A D</b>	<b>F</b>	<b>E</b>	<b>G</b>	<b>x</b>	<b>= 85 min</b>			

**VRAAG 6**

6.1 (a)  $n - 1$

(b)  $n(n - 1)$

6.2 (a)  $ABDCA \quad ACDBA$  }  
 $ACBDA \quad ADBCA$  }

(b)  $(n - 1)!$

**Totaal vir Module 4: 100 punte**

**Totaal: 100 punte**