GRADE 12 EXAMINATION
NOVEMBER 2016

ADVANCED PROGRAMME MATHEMATICS: PAPER II

Time: 1 hour 100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 14 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.

2. This question paper consists of THREE modules:

   Choose **ONE** of the **THREE** modules:

   MODULE 2: STATISTICS (100 marks) OR
   MODULE 3: FINANCE AND MODELLING (100 marks) OR
   MODULE 4: MATRICES AND GRAPH THEORY (100 marks)

3. Non-programmable and non-graphical calculators may be used.

4. All necessary calculations must be clearly shown and writing should be legible.

5. Diagrams have not been drawn to scale.

6. **Rounding of final answers.**

   MODULE 2: Four decimal places, unless otherwise stated.
   MODULE 3: Two decimal places, unless otherwise stated.
   MODULE 4: Two decimal places, unless otherwise stated.

MODULE 2 STATISTICS

QUESTION 1

1.1 In a large city one person in five is left-handed.

(a) Calculate the probability that in a random sample of 10 people, exactly three will be left-handed. (5)

(b) Find the most likely number of left-handed people in a random sample of 15 people. (2)

(c) In a random sample of n people, the probability that the sample contains at least one left-handed person is greater than 0,95. Solve for n, and hence determine the least number of people required in the sample. (8)

1.2 Bjorn saves his digital images on his computer in three separate folders, namely 'Family', 'Friends' and 'Rowing'. His Family folder contains three images, his Friends folder contains four images and his Rowing folder contains eight images. All the images are different.

(a) How many ways can he arrange these 15 images in a row across his computer screen if he keeps the images from each folder together? (5)

(b) Calculate the probability that if Bjorn chooses six images at random, there are two from each folder. (5)

(c) Calculate the number of different ways in which Bjorn can choose six of these images if there are at least three images from the Rowing folder and at least one image from each of the other two folders. (8)

[33]

QUESTION 2

2.1 It is known that the wind causes a 'chill factor' so that the human body experiences the temperature to be lower than the actual temperature. The following table gives the experienced temperature (t °C) for different wind speeds (w km/h) when the actual temperature is 10 °C. **(Work accurately to two decimal places in this question.)**

w (km/h)	0	5	10	15	20	25
t (°C)	10°	6°	–6°	–16°	–22°	–25°

(a) Determine the equation of a suitable regression line from which a value of t can be estimated for a given value w. (4)

(b) Find the value of the correlation coefficient for the data, and state what this indicates about the data. (2)

(c) Estimate the experienced temperature when the wind speed is 35 km/h. (2)

(d) Comment on the reliability of this estimate. (2)

2.2 Every day Timello tries to phone his friend Nicky. Every time he phones there is a 50% chance that Nicky will answer. If Nicky answers, Timello does not phone again on that day. If Nicky does not answer, Timello tries to phone again in a few minutes' time. If Nicky has not answered after four attempts, Timello does not try again on that day.

(a) Draw a tree diagram to illustrate this situation. (4)

(b) Let X be the number of unanswered phone calls made by Timello on a given day. Copy and complete the table, showing the probability distribution of X.

x	0	1	2	3	4
$P(X=x)$		$\frac{1}{4}$			

(4)

(c) Calculate the expected number of unanswered phone calls on a day if the formula for the Expected Value of the random variable X is given as:

$$E\left[X\right] = \sum P(X = x)\cdot x$$ (2)

[20]

QUESTION 3

3.1 In a survey of cars parked at a hockey match, 200 cars out of a random sample of 250 cars have been fitted with a tracking device.

(a) Calculate a 96% confidence interval for the population proportion of parked cars fitted with a tracking device. (6)

(b) Describe, in words, what this confidence interval means statistically. (2)

3.2 A random variable X is normally distributed with a mean μ and a standard deviation σ.

(a) Given that $5\sigma = 3\mu$, find $P(X < 2\mu)$. (8)

(b) Given that the standard deviation, σ, is 2 and that $P\left(X > \frac{1}{3}\mu\right) = 0{,}8$.

Calculate the mean, μ. (7)

[23]

QUESTION 4

Last year Gareth found that his journey to work took on average 45,7 minutes with a standard deviation of 3,2 minutes. Gareth wishes to test whether his travel time has increased this year. He notes the time, in minutes, for a random sample of eight journeys this year, with the following results.

46,2	41,7	49,2	47,1	47,2	48,4	53,7	45,5

It may be assumed that the population of this year's journey times is normally distributed also with a standard deviation of 3,2 minutes.

4.1 State, with a reason, whether Gareth should use a one-tailed or a two-tailed hypothesis test. (2)

4.2 Determine whether the sample provides significant evidence, at the 4% level of significance, that Gareth's travel time has increased this year. Specifically mention the conclusion of Gareth's findings. (10)

[12]

QUESTION 5

Three friends, Arnie, Michael and Connor go to a music concert, but they did not discuss at which entrance of the concert they will meet. There are four entrances: 1, 2, 3 and 4. Each friend chooses an entrance **independently**.

- The probability that Arnie chooses Entrance 1 is $\frac{1}{2}$ and equal probabilities for the remaining three entrances.
- Michael is equally likely to choose each of the four entrances.
- The probability that Connor chooses entrances 1, 2, 3 and 4 forms the ratio 1 : 1 : 2 : 3.

5.1 Calculate the probability that at least two friends will choose Entrance 1. (6)

5.2 Calculate the probability that the three friends will all meet at the same entrance. (6)

[12]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

Wynand inherits R42 000 from his grandmother, which he invests in a savings account. Eight months after this investment, he withdraws R5 000 to pay for repairs on his car. Fifteen months after his initial investment, his company gives him a bonus of R27 000, which he immediately invests in the same account. The account is open for a total of two years.

For the first year, the account earns interest at 4,62% per annum, compounded monthly. At the start of the second year, the interest is changed to 4,8% per annum, compounded quarterly.

1.1 Draw a timeline that represents the above situation. Indicate all changes relating to money and interest rates. Clearly indicate the time at which these changes occur. (6)

1.2 Calculate the final balance of the account at the end of the two years. (10)

[16]

QUESTION 2

Lolwethu inherits a R1 000 000 from a long-lost, wealthy relative! To supplement her income she decides to invest the money in a living annuity at 6,4% per annum, compounded quarterly.

She will withdraw R30 000 per quarter from the living annuity, starting immediately. Determine, in years and months, how long her inheritance will last.

[12]

QUESTION 3

Wayne's parents are planning for his final years of schooling in 2033, 2034 and 2035. They project the annual fees to be R28 000, R30 400 and R33 000 for the three years respectively.

The first payment into a savings account is to be made on 1 January 2017 and the last payment on 1 January 2035. They plan to deposit R270 each month into an account that earns 5,2% interest per annum, compounded monthly. Money will be withdrawn on 1 January 2033, 1 January 2034 and 1 January 2035 to pay for his fees.

3.1 Show that they assumed the annual (compound) inflation rate would be more or less constant during 2033 and 2034. (6)

3.2 Calculate the total value to which their monthly payments will accrue on 1 January 2035. (6)

3.3 Determine the total interest earned by the savings account over the full period of the investment. (4)

3.4 Determine whether the savings account will be sufficient to cover the annual fees for all three years. Show your calculations. (8)

[24]

QUESTION 4

4.1 Calculate the values of the next three terms in the sequence described by the second order recursive formula:

$$T_{n+1} = 4.T_n + 3.T_{n-1} - 4(-1)^{n-1} \text{ with } T_1 = -T_2 = -2$$ (5)

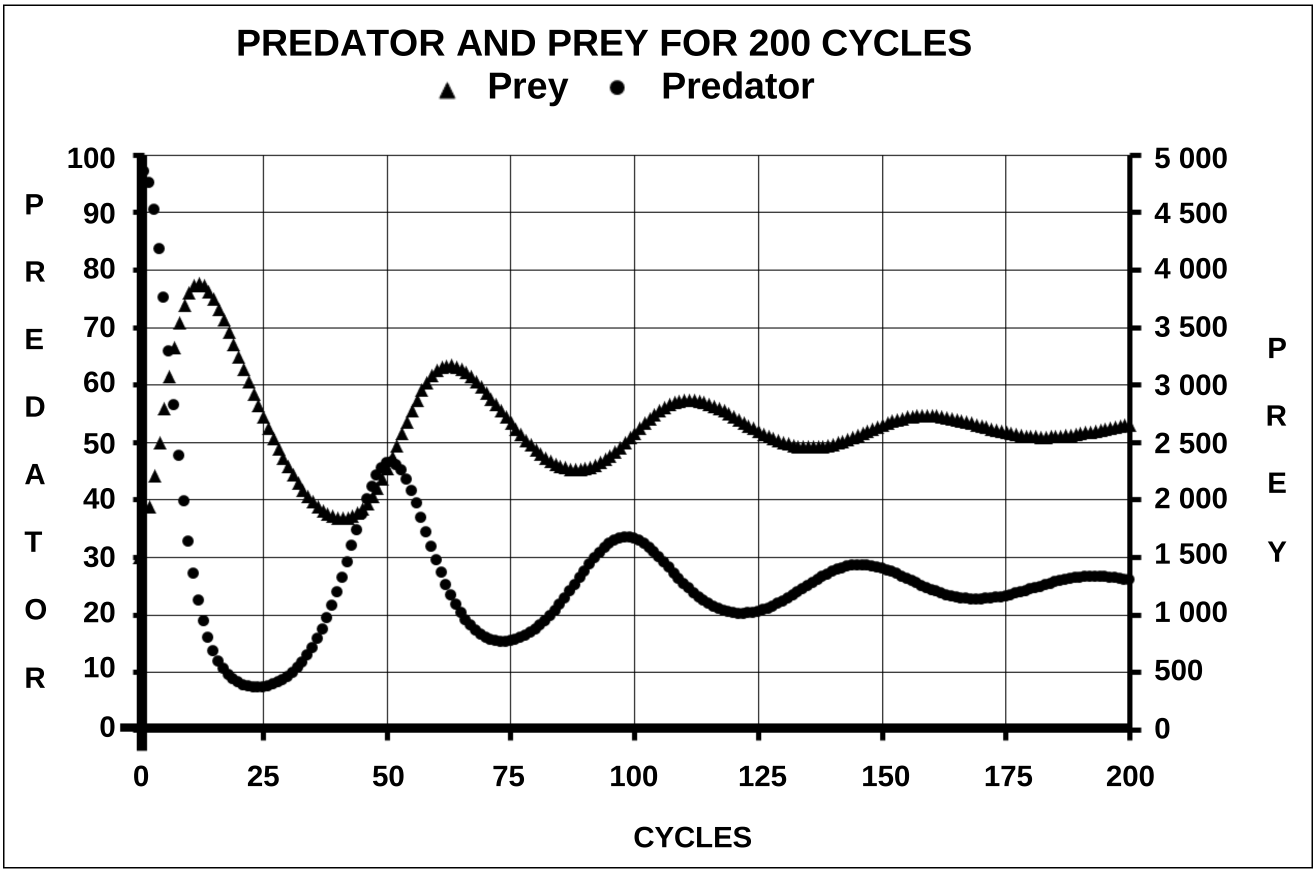
4.2 A Malthusian model is described by the recursive formula $T_{n+1} = p.T_n$ with $0 < p < 1$. Draw a graph of T_n against n (with n the independent variable) that represents the population trend for this model as n increases. (4)

4.3 A Logistic model has a carrying capacity of 120 and an initial population of 54. **Two** cycles later the population has reached 70. Calculate the intrinsic growth rate of the model, correct to two decimal digits. (9)

[18]

QUESTION 5

The accompanying graph represents 200 cycles of the populations for a particular predator and prey relationship, based on the Lotka-Volterra model.



5.1 This is a discrete population model. How can this be seen in the graph, and what does 'discrete' mean mathematically? (2)

5.2 Give the approximate range of the prey population as an interval. (2)

5.3 Approximately, how many cycles after each peak of the prey population does the predator population peak? (2)

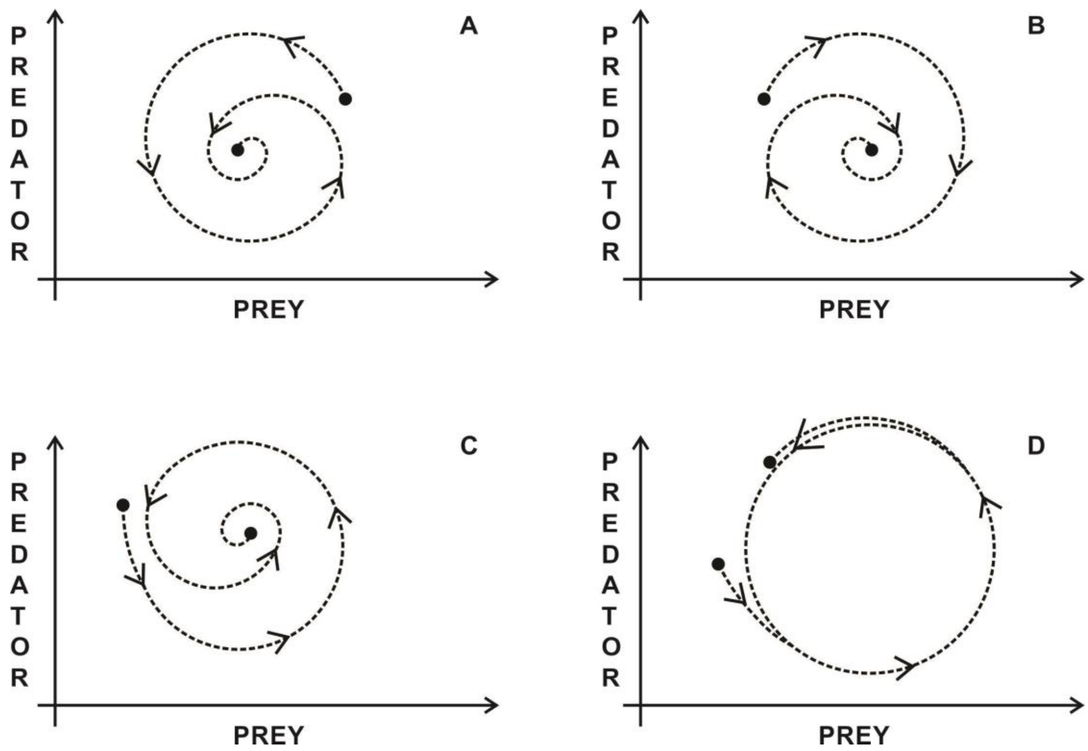
5.4 In the predator population, which is the greater: the rate of increase from a minimum to a maximum, or the rate of decrease from a maximum to a minimum? Give a reason for your answer. (2)

5.5 Refer to the Lotka-Volterra formulae in the **Information Booklet**:

(a) Which of the parameters *a*, *b* or *K* need to be decreased so that the prey population increases? Give a reason for your answer. (3)

(b) The parameter *c* is increased. What impact could this have on the prey population? Explain your answer fully in words. (3)

5.6 Which of the phase plots best represent this particular model? Explain your choice.

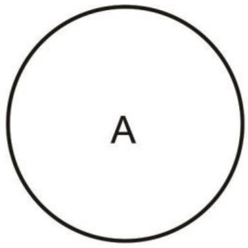


(4)

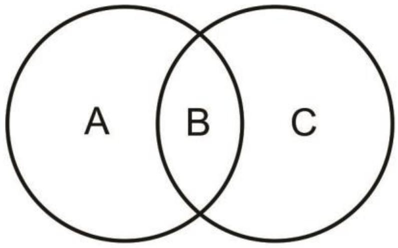
[18]

QUESTION 6

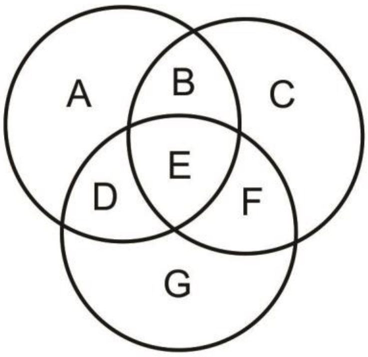
Venn diagrams are a convenient way of representing the intersections of sets. The number of regions they form with each other creates a numerical pattern.



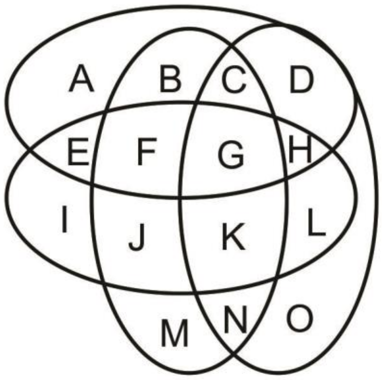
For one set, there is only one internal region (A)



For two sets, there are three internal regions (A, B and C)



For three sets, there are seven internal regions (A to G)



For 4 sets, there are 15 internal regions (A to O)

6.1 When four sets are used, give the letter of the region that represents the intersection of all four sets. (2)

6.2 To draw five intersecting sets would be unreasonable. However, how many internal regions would you expect five intersecting sets to have? (2)

6.3 Create a first order recursive formula that will generate the number of internal regions with the addition of each new set. (4)

6.4 Determine the maximum number of sets used before reaching one million internal regions. (4)

[12]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 Given the matrix equation: $\begin{pmatrix} 1 & -1 \\ 2 & 4 \\ x & y \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 10 & z \\ 4 & -11 \end{pmatrix}$

Calculate the values of x, y and z. (8)

1.2 For each of the following matrix identities, assume the operations are possible. State if the following identities are necessarily TRUE or possibly FALSE:

(a) AB = BA (2)

(b) AB = AC implies that B = C (2)

(c) $A^{-1} \cdot A = A \cdot A^{-1} = I$ (2)

(d) A(BC) = (AB)C (2)

[16]

QUESTION 2

2.1 Consider the transformation represented by the matrix equation:

$$\begin{pmatrix} 1 & -1 & 2 \\ 4 & 3 & 0 \end{pmatrix} + \begin{pmatrix} +3 & +3 & +3 \\ -1 & -1 & -1 \end{pmatrix} = M$$

(a) Describe this transformation in words. (3)

(b) Is this a rigid transformation? (1)

(c) Write down matrix M, the image of the figure after the transformation. (2)

2.2 Consider the transformation represented by the matrix equation:

$$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 4 & 3 & 0 \end{pmatrix} = N$$

(a) Describe this transformation in words. (3)

(b) Is this a rigid transformation? (1)

(c) Which number related to a transformation matrix gives the scale factor by which the area of the figure must be multiplied in order to determine the area of its image? (1)

(d) Calculate the number mentioned in Question 2.2 (c). Explain the numerical relationship between the area of this figure and the area of its image. (3)

2.3 A figure in a Cartesian plane is to be rotated 30° anti-clockwise about the origin, followed by a reflection in the line $y = 3x$. Determine a single matrix that would produce these transformations in the stated order. Give the elements of this matrix correct to two decimal places. (8)

[22]

QUESTION 3

Karyn needs to solve three linear equations simultaneously for the variables x, y and z. In order to do this, she designs the following matrix equation:

$$\begin{pmatrix} 6 & 2 & -3 \\ 6 & 3 & 1 \\ 9 & 3 & t \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 15 \\ w \end{pmatrix}$$

3.1 Karyn realises that for $t = -2$ and $w = 8$ the matrix equation has a unique solution. Determine this solution through calculation. Do **not** merely state the solution; show at least one line of working using the given matrix equation. (8)

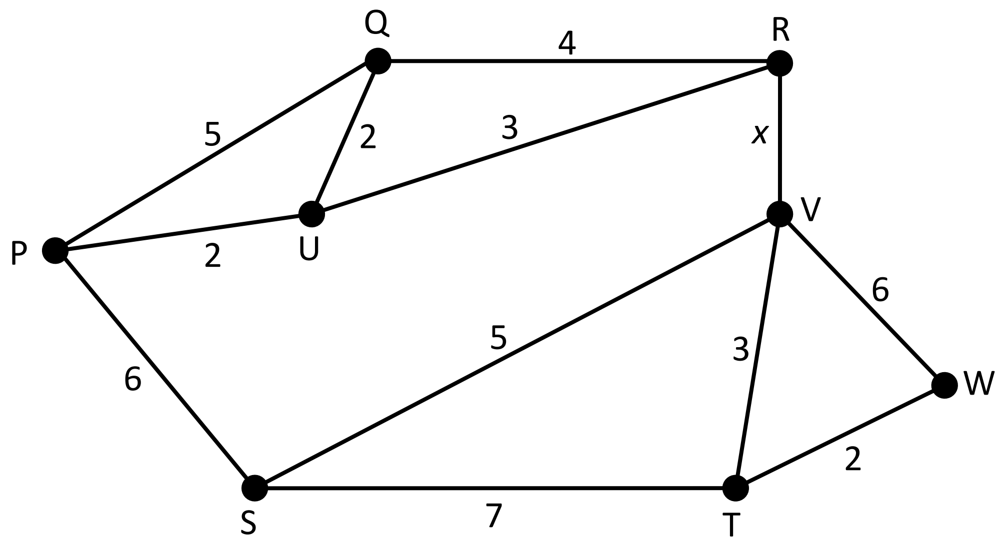
3.2 Calculate the value of t for which the matrix equation does **not** have a unique solution, irrespective of the value of w. (6)

3.3 Upon further investigation, Karyn realises that using the value of t obtained in Question 3.2, the matrix equation could in fact have an infinite number of solutions. State the corresponding value of w for this to be true. (2)

[16]

QUESTION 4

In the graph below, the path of least weight between vertex P and vertex W is to be determined. The weight of edge RV is unknown and represented by *x*.



4.1 In the circuit PUQ, edge PQ can be ignored as PQ > PU + QU. What other edge in the graph can be ignored for a similar reason? (2)

4.2 Why can the weights of the edges **not** represent linear distances? (2)

4.3 Design a Hamiltonian circuit on this graph, starting at W. (4)

4.4 Calculate the maximum integer weight of RV so that P → U → R → V → T → W becomes the path of least weight. (6)

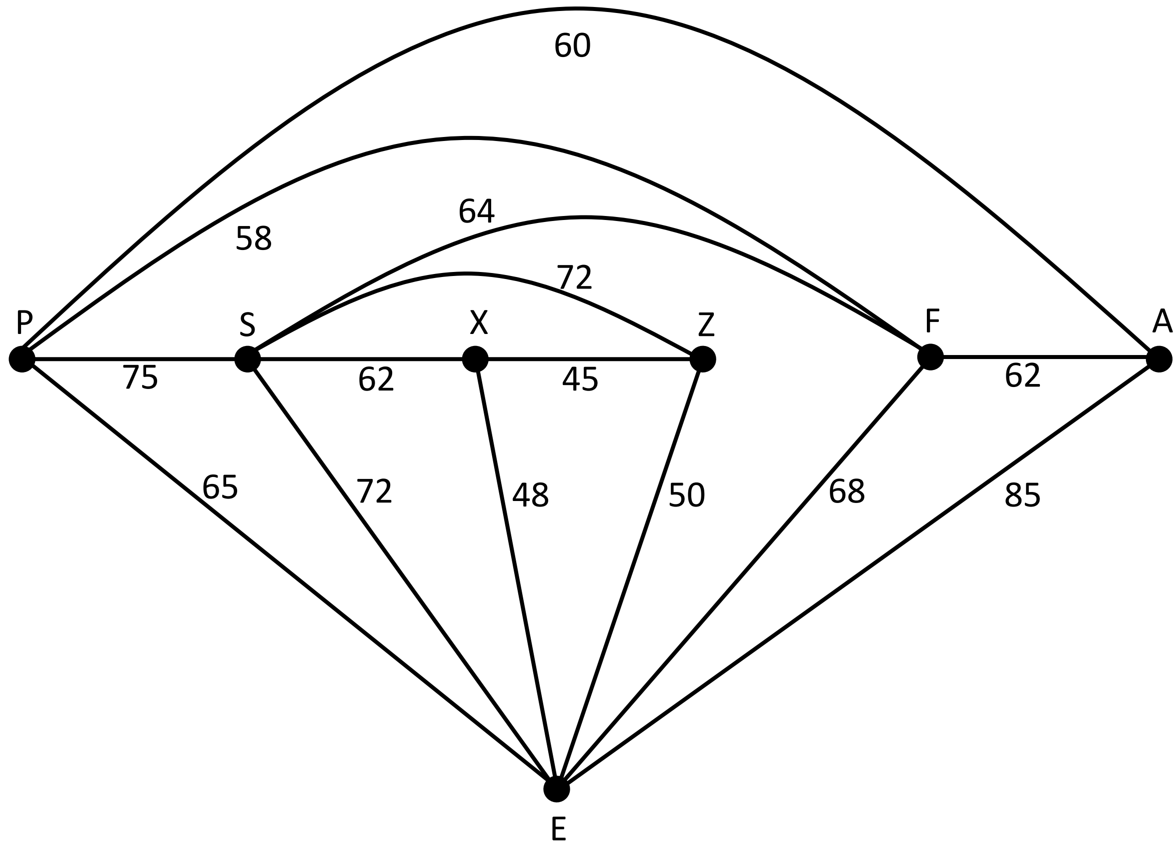
[14]

QUESTION 5

At African Union summits, five official languages are used: Arabic (A), English (E), French (F), Portuguese (P) and Swahili (S). South African delegates also use isiXhosa (X) and isiZulu (Z).

Documents need to be translated from their original language into each of the other six languages, and then back into the original language to check for inaccuracies.

In the graph below, languages are represented by the vertices. Each edge represents a direct translation between the two languages joined by the edge. The weight of the edges represents the average time taken in minutes for translation.



5.1 Which two of the seven languages are the most versatile for translation? (2)

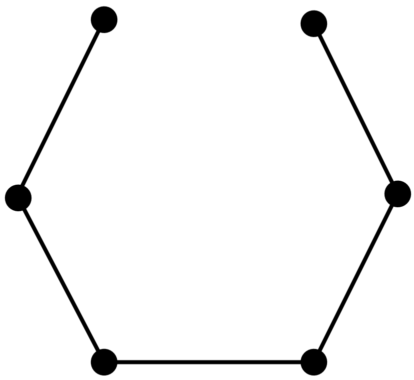
5.2 Starting at English, determine an upper bound for the time needed to translate a document into all other languages. Use the Nearest Neighbour Algorithm and clearly record the order in which edges are selected. (8)

5.3 Starting at English, use inspection to determine a 'good route' for the time needed for translation. Your solution should be at least 20 minutes quicker than the upper bound calculated in Question 5.2. (10)

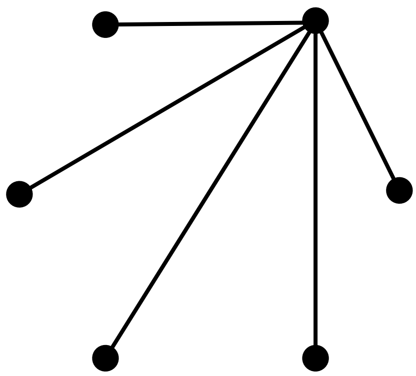
[20]

QUESTION 6

Six non-isomorphic spanning trees can be created on six vertices. Two are sketched below.



The vertices have degrees
1, 1, 2, 2, 2 and 2



The vertices have degrees
1, 1, 1, 1, 1 and 5

6.1 How many edges will each of the six spanning trees have? (2)

6.2 In each of these six cases, what is the sum of the degrees of the vertices? (2)

6.3 Sketch the four remaining non-isomorphic spanning trees on six vertices. (8)
[12]

Total for Module 4: 100 marks

Total: 100 marks