basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL/*NASIONALE*
SENIOR
CERTIFICATE/*SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2017

MEMORANDUM

MARKS / PUNTE: 150

This memorandum consists of 21 pages.
Hierdie memorandum bestaan uit 21 bladsye.

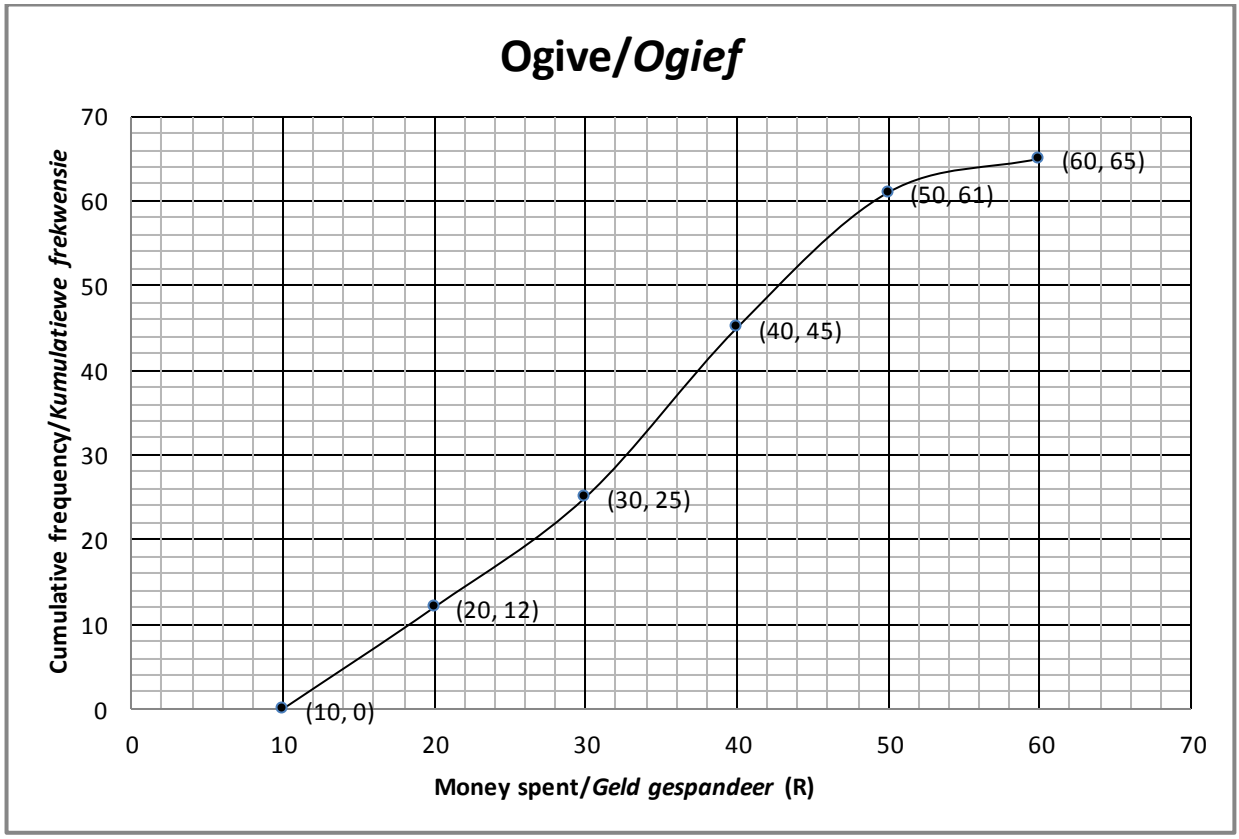
NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

QUESTION/*VRAAG* 1

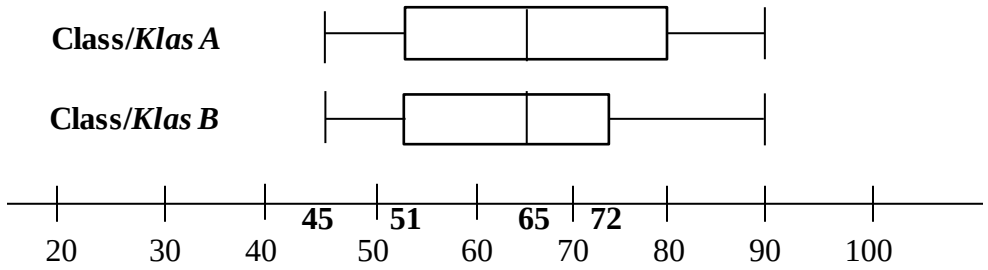


Amount of money/ *Bedrag geld* (in R)	$10 \le x < 20$	$20 \le x < 30$	$30 \le x < 40$	$40 \le x < 50$	$50 \le x < 60$
Frequency *Frekwensie*	*a*	13	20	*b*	4

1.1	65 learners/*leerders*	✓ answer (1)
1.2	Modal class/*Modale klas*: $30 \le x < 40$	✓ answer (1)
1.3	$a = 12$ $b = 61 - 45$ $= 16$	✓ answer ✓ answer (2)
1.4	No. of learners/*Aantal leerders* = 65 – 54 **OR/*OF*** 65 – 55 = 11 = 10 Answer only: full marks	✓ 54 or 55 ✓ 11 or 10 (2) **[6]**

QUESTION/*VRAAG* 2

2.1



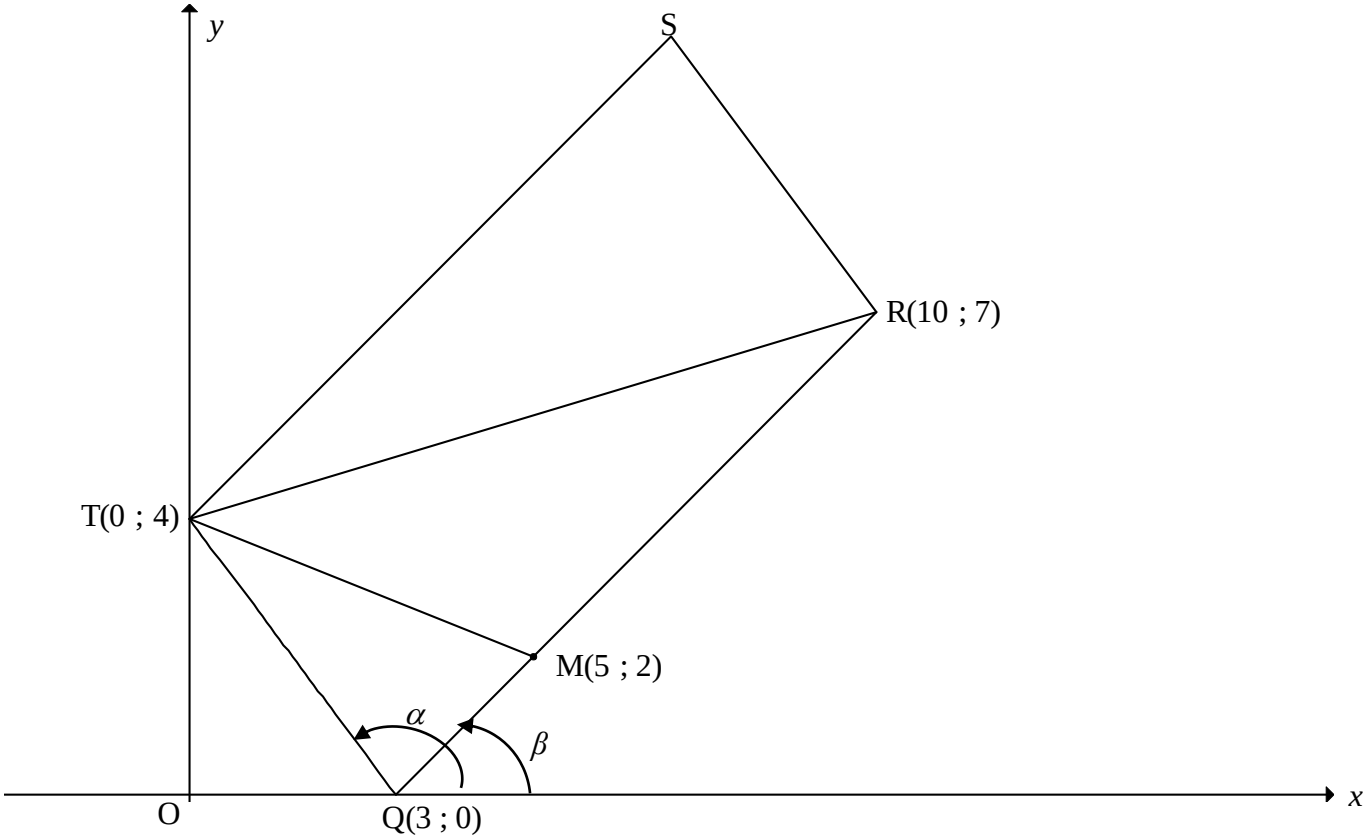
2.1.1	IQR of Class B/*IKV van Klas B* = $Q_3 - Q_1$ = 72 – 51 = 21 marks/*punte*	✓72 and 51 ✓ 21 only (2)
2.1.2	Although the boxes contain the same number of data points, the marks for Class A are more widely spread./*Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas A meer verspreid.* **OR/OF** Although the boxes contain the same number of data points, the marks for Class B are more clustered./*Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas B nader aan mekaar.*	✓ ✓ Class A is more widely spread (2) ✓ ✓ Class B is more clustered (2)
2.1.3	Medians are the same/*Mediane is dieselfde* Ranges are the same **OR** Maximum and minimum values are the same/*Variasiewydtes is dieselfde **OF** die maksimum en minimum waarde is dieselfde* 75% of both classes obtained 51 and above/*75% van albei klasse behaal 51 en meer.*	✓ ✓ any TWO of the 3 reasons mentioned (2)

2.2

COUPLE/*PAAR*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
JUDGE 1/ *BEOORDELAAR 1*	18	4	6	8	5	12	10	14
JUDGE 2/ *BEOORDELAAR 2*	15	6	3	5	5	14	8	15

2.2.1	$a = -0,03$ $b = 0,93$ $\hat{y} = -0,03 + 0,93x$	✓ value a ✓ value b ✓ equation (3)
2.2.2	$\hat{y} = -0,03 + 0,93(15)$ = 13,92 **OR/*OF*** 13,85 ≈ 14	✓ substitution ✓ answer (2)
2.2.3	Yes **OR** they are consistent, because $r = 0,9$. ($r = 0,89567\ldots$)/*Ja OF hulle is konsekwent, want $r = 0,9$. ($r = 0,89567\ldots$)*	✓ statement ✓ $r = 0,9$ (2) **[13]**

QUESTION/*VRAAG* 3



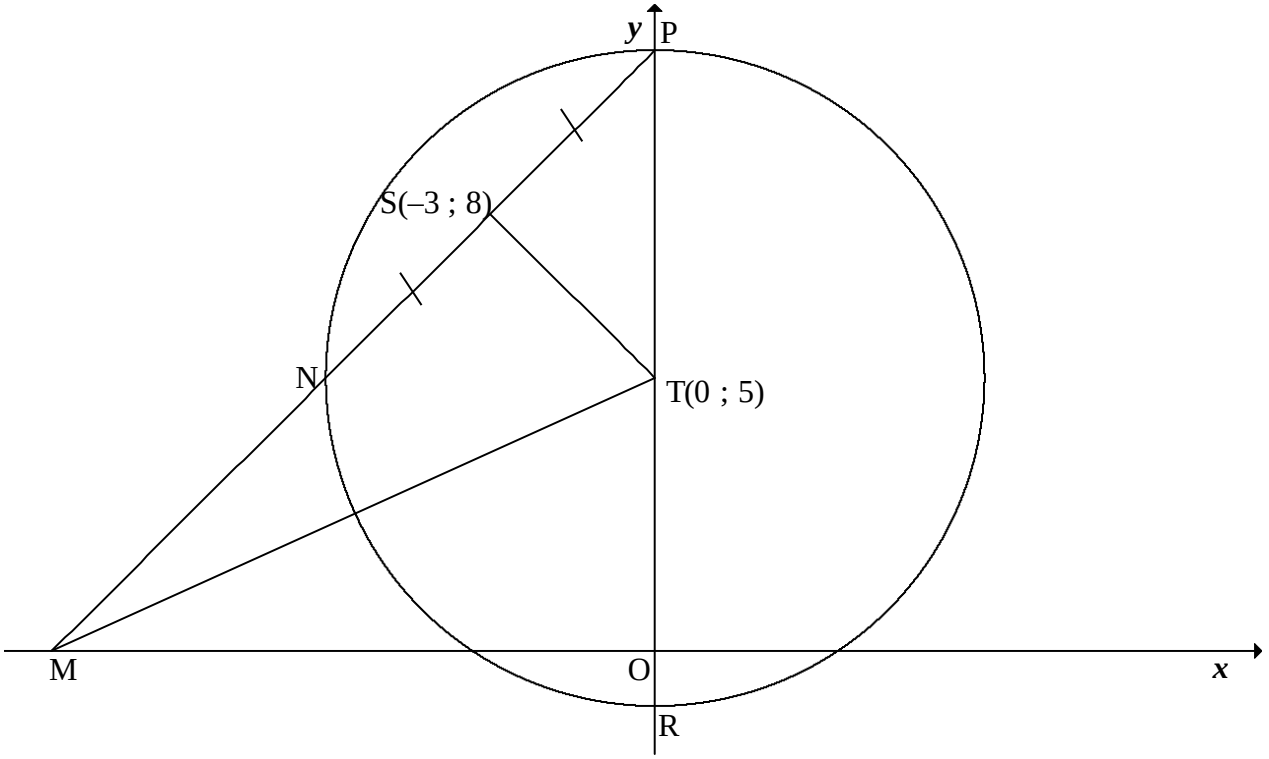
3.1	$m_{TQ} = \frac{4-0}{0-3}$ $= -\frac{4}{3}$	✓ answer (1)
3.2	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $RQ = \sqrt{(10-3)^2 + (7-0)^2}$ $RQ = \sqrt{98} = 7\sqrt{2}$	✓substitution/*substitusie* ✓ answer in surd form (2)
3.3	$m_{FQ} = m_{TQ}$ $\frac{-8}{k-3} = -\frac{4}{3}$ $4k - 12 = 24$ $k = 9$ **OR/*OF*** $m_{FT} = m_{QT}$ $\frac{-8-4}{k-0} = -\frac{4}{3}$ $-36 = -4k$ $k = 9$ **OR/*OF*** Equation of TQ: $y = -\frac{4}{3}x + 4$ $-8 = -\frac{4}{3}k + 4$ $k = 9$	✓ equating gradients/*stel gradient gelyk* ✓ $m_{FQ} = \frac{-8}{k-3}$ ✓ simplification/*vereenvoudig* ✓ answer (4) ✓ gradient ✓ equation of TQ/*vgl van TQ* ✓ substitution of $(k\ ;\ -8)$ /*substitusie van* $(k\ ;\ -8)$ ✓ answer (4)

3.4	Using transformation/*Gebruik transformasie*: $\therefore$ S(7 ; 11) **OR/*OF*** Midpoint of TR = midpoint of SQ [diag ||m/*hkle*||*m*] Midpoint of TR = $(5 ; \frac{11}{2})$ $\frac{x_S + 3}{2} = 5$ and $\frac{y_S + 0}{2} = \frac{11}{2}$ $\therefore x_S = 7$ and $y_S = 11$ $\therefore$ S(7 ; 11) **OR/*OF*** Equation of TS: $y = \left(\frac{7-2}{10-5}\right)x + 4 = x + 4$ Equation of RS: $y - 7 = -\frac{4}{3}(x - 10)$ $y = -\frac{4}{3}x + \frac{61}{3}$ $x + 4 = -\frac{4}{3}x + \frac{61}{3}$ $7x = 49$ $x = 7$ $\therefore \quad y = 11$ $\therefore$ S(7 ; 11)	✓ ✓ *x*–value/*waarde* ✓ ✓ *y*–value/*waarde* (4) ✓ *x*–value/*waarde* of/*van* T ✓ *y*–value/*waarde* of/*van* T ✓ *x*–value/*waarde* of/*van* S ✓ *y*–value/*waarde* of/*van* S (4) ✓ equations of TS and RS/*vgls van TS en RS* ✓ equating / *gelykstel* ✓ *x*–value/*waarde* ✓ *y*–value/*waarde* (4)
3.5	$T\hat{S}R = T\hat{Q}R$ [opp $\angle$s of ||m/*teenoorst* $\angle e$ ||*m*] $T\hat{Q}R = \alpha - \beta$ $tan\,\alpha = m_{TQ} = -\frac{4}{3}$ $\therefore \alpha = 180° - 53{,}13° = 126{,}87°$ $tan\,\beta = m_{RQ} = \frac{7}{7} = 1$ $\therefore \beta = 45°$ $T\hat{Q}R = 126{,}87° - 45°$ $= 81{,}87°$ $T\hat{S}R = 81{,}87°$ **OR/*OF***	✓ $T\hat{Q}R = \alpha - \beta$ ✓ $\tan\alpha = m_{TQ}$ ✓ α ✓ $\tan\beta = m_{RQ}$ ✓ β ✓ answer (6)

	TQ = SR = 5 TR = $\sqrt{100+9}=\sqrt{109}$ RQ = TS = $\sqrt{49+49}=\sqrt{98}$ $\cos R\hat{Q}T = \cos T\hat{S}R = \frac{TQ^2+RQ^2-TR^2}{2.TQ.RQ}$ $= \frac{25+98-109}{2(5)(\sqrt{98})}$ $= 0{,}141...$ $R\hat{Q}T = T\hat{S}R = 81{,}87°$	✓ length of TQ **OR** SR ✓ length of TR ✓ length of RQ **OR** TS ✓ correct subst into cosine rule ✓ simplification ✓ answer (6)
3.6.1	$MQ=\sqrt{(5-3)^2+(2-0)^2}$ $MQ=\sqrt{8}$ $\frac{MQ}{RQ}=\frac{\sqrt{8}}{\sqrt{98}}$ Answer only: full marks $=\frac{2}{7}$ or 0,29	✓substitution/*substitusie* ✓ $MQ=\sqrt{8}=2\sqrt{2}$ ✓ answer (3)
3.6.2	$\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR}=\frac{\frac{1}{2}.QM.\perp h}{\frac{1}{2}.QR.\perp h}$ [⊥h same/*dieselfde*] $=\frac{QM}{QR}=\frac{2}{7}$ $\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS}=\frac{\text{area of } \Delta TQM}{2\times\text{area of } \Delta TQR}$ $=\frac{1}{2}\left(\frac{2}{7}\right)=\frac{1}{7}$ **OR/*OF*** $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR}=\frac{QM}{QR}$ $=\frac{2}{7}$ $\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS}=\frac{\text{area of } \Delta TQM}{2\text{area of } \Delta TQR}$ $=\frac{1}{2}\left(\frac{2}{7}\right)=\frac{1}{7}$ **OR/*OF***	✓ $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR}=\frac{2}{7}$ ✓ area parm $RQTS = 2\text{area } \Delta TQR$ ✓ answer (3) ✓ $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR}=\frac{2}{7}$ ✓ area parm $RQTS = 2\text{area } \Delta TQR$ ✓ answer (3)

	$\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\frac{1}{2}QM.\perp h}{RQ.\perp h}$ $= \frac{1}{2}\left(\frac{2}{7}\right)$ $= \frac{1}{7}$ **OR/*OF*** $\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\frac{1}{2}QT.QM.\sin(\alpha-\beta)}{2\text{area of } \Delta QTR}$ $= \frac{\frac{1}{2}QT.QM.\sin(\alpha-\beta)}{2[\frac{1}{2}.QT.QR.\sin(\alpha-\beta)]}$ $= \frac{1}{2}\left(\frac{2}{7}\right)$ $= \frac{1}{7}$	✓ $\frac{\frac{1}{2}QM.\perp h}{RQ.\perp h}$ ✓ $\frac{1}{2}\left(\frac{2}{7}\right)$ ✓ answer (3) ✓ area parm $RQTS = 2\text{area } \Delta TQR$ ✓ $\frac{\frac{1}{2}QT.QM.\sin(\alpha-\beta)}{2[\frac{1}{2}.QT.QR.\sin(\alpha-\beta)]}$ ✓ answer (3) **[23]**

QUESTION/*VRAAG* 4



4.1	line from centre to **midpt of chord** / *lyn vanaf midpt na midpt van koord*	✓ answer (1)
4.2	$m_{ST} = \frac{8-5}{-3-0}$ $= -1$ $m_{ST} \times m_{NP} = -1$ [TS ⊥ NP] $\therefore\ m_{NP} = 1$ $\therefore y = x + c$ $8 = -3 + c$ $c = 11$ $\therefore y = x + 11$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 8 = 1(x + 3)$ $y = x + 11$	✓ subst (–3 ; 8) and (0 ; 5) into gradient formula ✓ m_{ST} ✓ m_{NP} ✓ subst (–3 ; 8) into equation of a line ✓ equation (5)
4.3	P(0 ; 11) [*y*-intercept of chord NP] ∴ radius is 6 units R(0 ; –1) Equations of the tangents to the circle parallel to the *x*-axis/ *Vgls van die raaklyne aan die sirkel || aan die x-as*: $y = 11$ and $y = -1$	✓ coordinates of P/ koördinate v P ✓ coordinates of R koördinate van R ✓✓ answers (4)
4.4	M(–11 ; 0) [*x*–intercept of/*x-afsnit van* NP] $MT = \sqrt{(0-11)^2 + (5-0)^2}$ $MT = \sqrt{146} = 12{,}08$	✓✓ coordinates of M ✓ substitution ✓ answer (4)

4.5	MT = diameter/*middellyn* [conv∠ in $\frac{1}{2}$ circle/*omgek ∠in $\frac{1}{2}$ sirkel*] radius = $\frac{\sqrt{146}}{2}$ units Centre of circle/*Middelpunt v sirkel* = Midpoint MT /*Middelpunt MT* = $\left(\frac{-11}{2};\frac{5}{2}\right)$ Equation of circle through S, T and M: $\left(x+\frac{11}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\frac{146}{4}$ **OR/*OF*** $\left(x+5\frac{1}{2}\right)^2+\left(y-2\frac{1}{2}\right)^2=\frac{73}{2}=6{,}04$	✓ radius of circle ✓ *x* value of M ✓ *y* value of M ✓ LHS of equation ✓ RHS of equation (5) **[19]**

QUESTION/*VRAAG* 5

5.1	$a=-1$ $b=2$	✓ answer ✓ answer (2)
5.2	$f(3x)=-\sin 3x$ Period of $f(3x)=\frac{360°}{3}$ $=120°$ Answer only: Full marks	✓ $\frac{360°}{3}$ ✓ answer (2)
5.3	$x \in [90° ; 135°) \cup \{180°\}$ **OR/*OF*** $90° \leq x < 135°$ or $x = 180°$	✓ 90° and 135° in interval form ✓ 180° as single value ✓ correct brackets (3) ✓ 90° and 135° in interval form ✓ 180° as single value ✓ correct inequalities (3) **[7]**

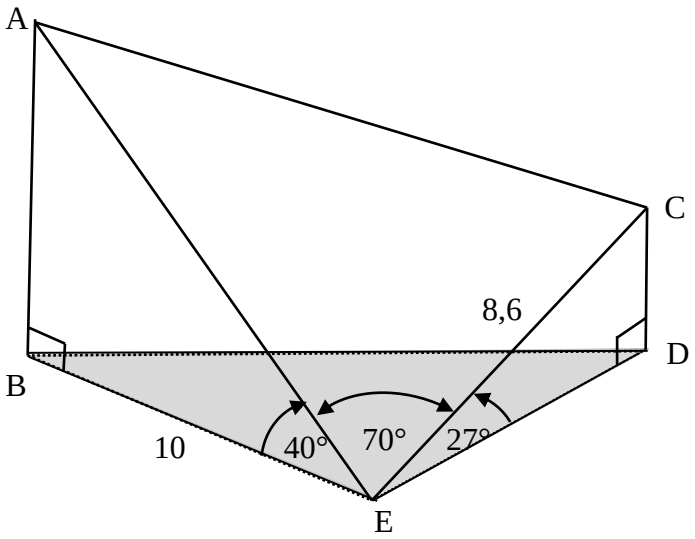
QUESTION/*VRAAG* 6

6.1.1	$\sin(360° - 36°) = -\sin 36°$	✓ answer (1)
6.1.2	$\cos 72° = \cos(2 \times 36°)$ $= 1 - 2\sin^2 36°$ Answer only: Full marks	✓ double angle/*dubbelhoek* ✓answer (2)
6.2	R.T.P.: $1 - \frac{\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta$ $\text{LHS} = \frac{1+\tan^2\theta-\tan^2\theta}{1+\tan^2\theta}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{\frac{1}{\cos^2\theta}}$ $= \cos^2\theta$ = RHS **OR/*OF*** $\text{LHS} = \frac{1+\tan^2\theta-\tan^2\theta}{1+\tan^2\theta}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}} \times \frac{\cos^2\theta}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta}$ $= \frac{\cos^2\theta}{1}$ $= \cos^2\theta$ = RHS **OR/*OF***	✓ writing as a single fraction/*skryf as enkelbreuk* ✓ quotient identity/ *kwosiëntidentiteit* ✓ denominator as a single fraction / *Noemer as enkelbreuk* ✓ square identity/*vierkantidentiteit* (4) ✓ writing as a single fraction/*skryf as enkelbreuk* ✓ quotient identity / *kwosiëntidentiteit* ✓ $\times \frac{\cos^2\theta}{\cos^2\theta}$ ✓ square identity/*vierkantidentiteit* (4) ✓ quotient identity/

	$\text{LHS} = 1-\left(\frac{\sin^2\theta}{\cos^2\theta} \div \left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\right)$ $= 1-\left(\frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta}\right)$ $= 1-\left(\frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{1}\right)$ $= 1-\sin^2\theta$ $= \cos^2\theta$ $= \text{RHS}$	*kwosiëntidentiteit* ✓ writing as a single fraction/ *skryf as enkelbreuk* ✓ square identity/*vierkantidentiteit* ✓ simplification/*vereenvoudiging* (4)
6.3	$\cos^2\frac{1}{2}x = \frac{1}{4}$ $\cos\frac{1}{2}x = \frac{1}{2}$ *or* $-\frac{1}{2}$ $\frac{1}{2}x = 60°+k.360°$ or $\frac{1}{2}x = 300°+k.360°$ or $\frac{1}{2}x = 120°+k.360°$ or $\frac{1}{2}x = 240°+k.360°$ $x = 120°+k.720°$ or $x = 600°+k.720°$ or $x = 240°+k.720°$ or $x = 480°+k.720°;\ k \in Z$ **OR/*OF*** $\cos^2\frac{1}{2}x = \frac{1}{4}$ $\cos\frac{1}{2}x = \frac{1}{2}$ *or* $-\frac{1}{2}$ $\frac{1}{2}x = \pm 60°+k.360°$ or $\frac{1}{2}x = \pm 120°+k.360°$ $x = \pm 120°+k.720°$ or $x = \pm 240°+k.720°;\ k \in Z$	✓✓ $\cos^2\frac{1}{2}x = \frac{1}{4}$ ✓ 60° and 300° ✓ 120° and 240° ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360°$ ✓ write at least one general solution as $x = \angle + k.720°;\ k \in Z$ (6) ✓✓ $\cos^2\frac{1}{2}x = \frac{1}{4}$ ✓ ±60° ✓±120° ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360°$ ✓ write at least one general solution as $x = \angle + k.720°\ k \in Z$ (6)

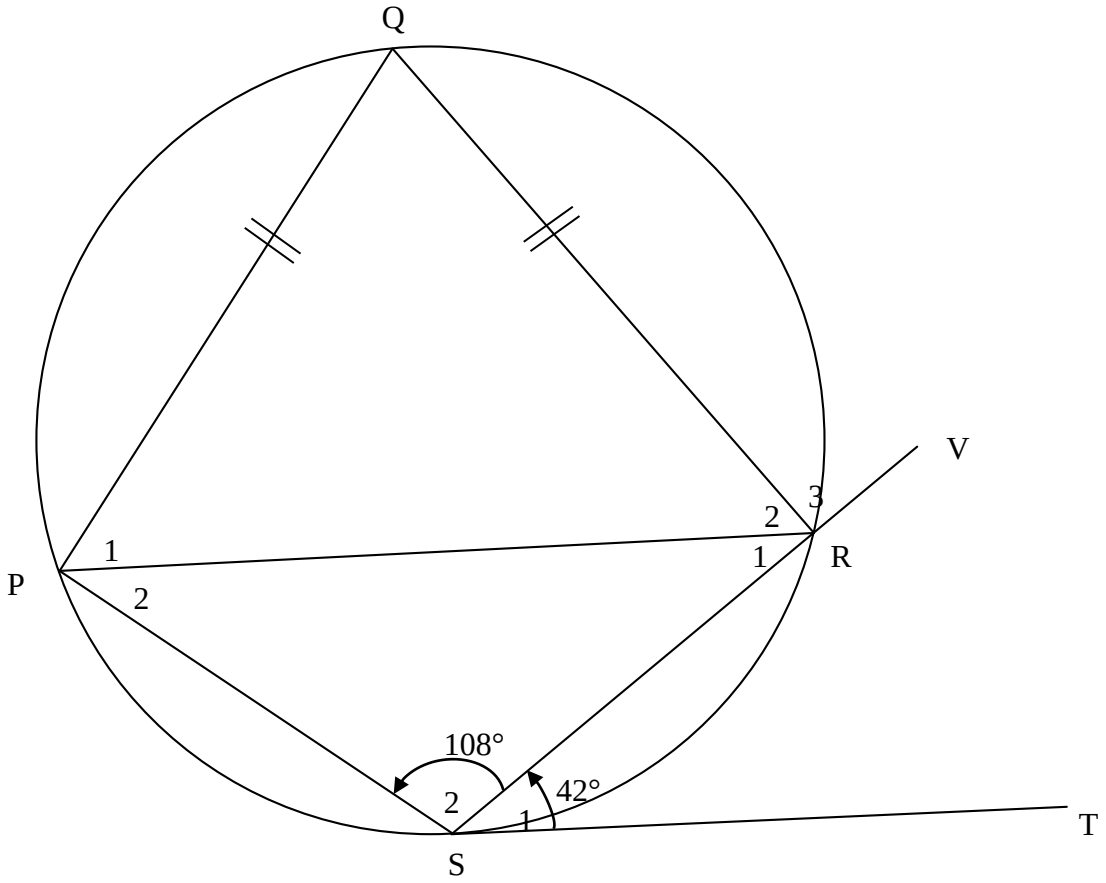
6.4.1	$\sin(A-B) = \cos[90°-(A-B)]$ $= \cos[(90°-A)-(-B)]$ $= \cos(90°-A)\cos(-B)+\sin(90°-A)\sin(-B)$ $= \sin A\cos B+\cos A(-\sin B)$ $= \sin A\cos B-\cos A\sin B$ **OR/*OF*** $\sin(A-B) = \cos[90°-(A-B)]$ $= \cos[(90°+B)-A]$ $= \cos(90°+B)\cos A+\sin(90°+B)\sin A$ $= -\sin B\cos A+\cos B\sin A$ $= \sin A\cos B-\cos A\sin B$	✓ co–ratio/*ko-verhouding* ✓ writing as a difference of A & B/ *skryf as verskil van A & B* ✓ expansion/*uitbreiding* ✓ all reductions/*alle reduksies* (4) ✓ co–ratio/*ko-verhouding* ✓ writing as a difference of A & B/ *skryf as verskil van A & B* ✓ expansion/*uitbreiding* ✓ all reductions/*alle reduksies* (4)
6.4.2	$\sin(x+64°)\cos(x+379°)+\sin(x+19°)\cos(x+244°)$ $= \sin(x+64°)\cos(x+19°)+\sin(x+19°)[-\cos(x+64°)]$ $= \sin(x+64°)\cos(x+19°)-\cos(x+64°)\sin(x+19°)$ $= \sin[x+64°-(x+19°)]$ $= \sin 45°$ $= \frac{1}{\sqrt{2}}$	✓ $\cos(x+379°) = \cos(x+19°)$ ✓✓ $\cos(x+244°) = -\cos(x+64°)$ ✓✓compound formula identity/ *saamgestelde identiteit* ✓ $\sin 45°$ (6) **[23]**

QUESTION/*VRAAG* 7



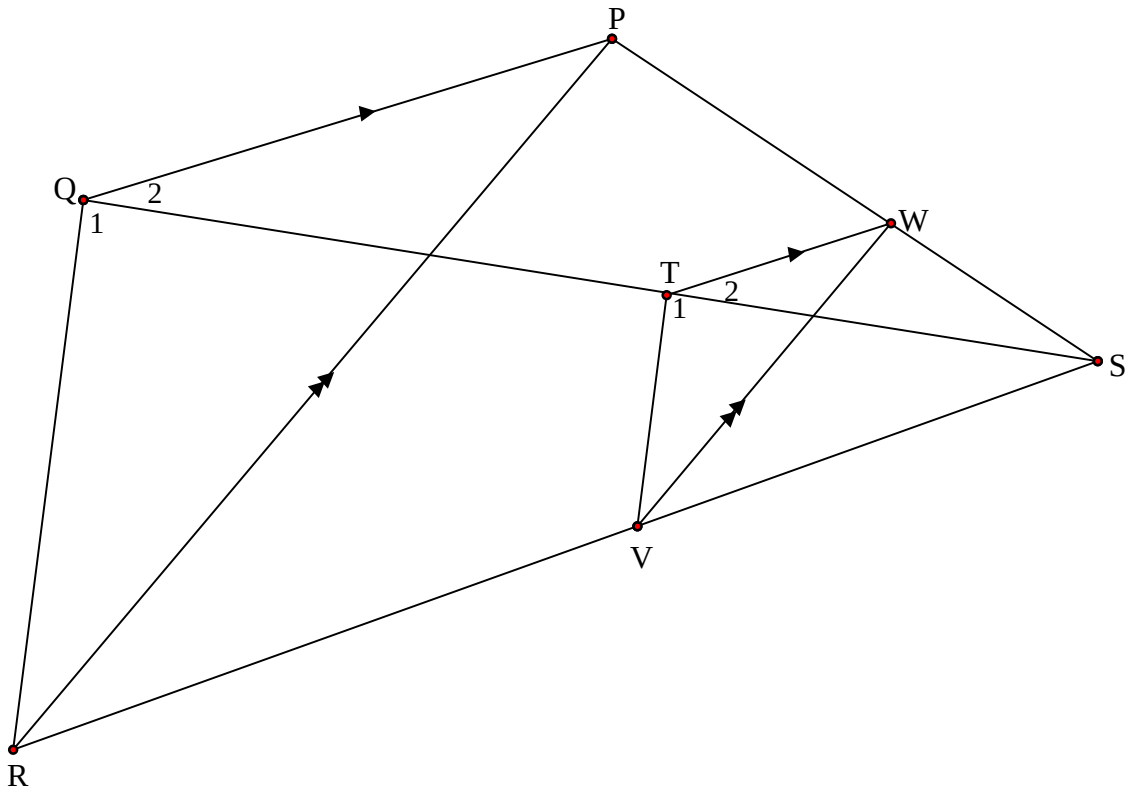
7.1	$\sin 27° = \frac{CD}{8,6}$ $CD = 8,6\sin 27°$ $CD = 3,90\text{ m}$	✓ substitution in correct trig ratio / *substitusie in korrekte trig verh* ✓ answer (2)
7.2	$\cos 40° = \frac{10}{AE}$ $AE = \frac{10}{\cos 40°}$ $AE = 13,05\text{ m}$	✓ substitution in correct trig ratio / *substitusie in korrekte trig verh* ✓ answer (2)
7.3	$AC^2 = CE^2 + AE^2 - 2\,CE.AE(\cos A\hat{E}C)$ $= (8,6)^2 + (13,05)^2 - 2(8,6)(13,05)(\cos 70°)$ $= 167,49$ $AC = 12,94\text{ m}$	✓ correct use of cosine rule in ΔACE/ *korrekte gebruik van reel in* ΔACE ✓ correct subst into cosine rule ✓ AC^2 ✓ answer (4) **[8]**

QUESTION/*VRAAG* 8



8.1	$\hat{Q} = 72°$ [opp ∠s of cyclic quad/*teenoorst ∠e koordevh*]	✓ S ✓ R (2)
8.2	$\hat{R}_2 = \hat{P}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\hat{R}_2 = \frac{180° - 72°}{2}$ [sum of ∠s in Δ/*som v ∠e in Δ*] $= 54°$	✓ S/R ✓ answer (2)
8.3	$\hat{P}_2 = 42°$ [tan chord theorem/*raakl-koordst*]	✓ S ✓ R (2)
8.4	$\hat{R}_3 = \hat{P}_1 + \hat{P}_2$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $= 54° + 42°$ $= 96°$ **OR/*OF*** $\hat{R}_1 = 180° - 108° - 42° = 30°$ [sum of/*som van*∠s/e in Δ] $\hat{R}_3 = 180° - \hat{R}_1 - \hat{R}_2$ [∠s on str line/*∠e op reguitlyn*] $= 180° - 30° - 54°$ [sum of/*som van*∠s/e in Δ] $= 96°$	✓ R ✓ S (2) ✓ $\hat{R}_1 = 30°$ ✓ S (2) **[8]**

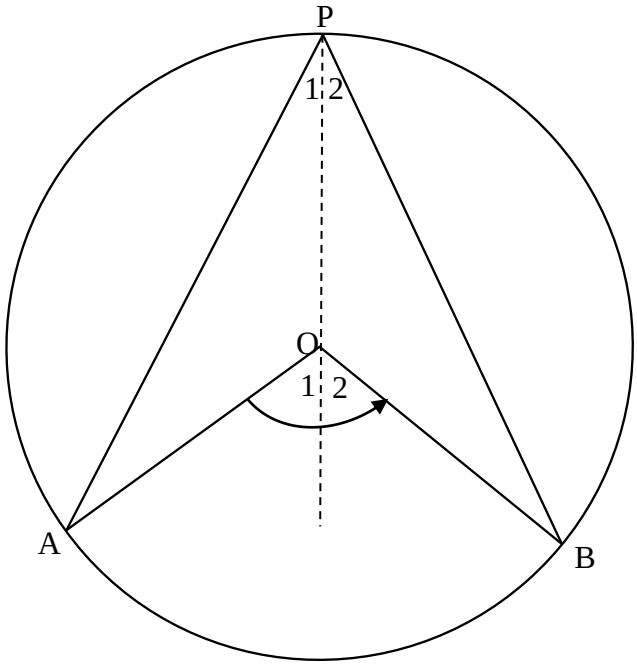
QUESTION/*VRAAG* 9



9.1.1	$\frac{ST}{TQ} = \frac{SW}{WP}$ [prop theorem/*eweredighst*; TW ‖ QP] $= \frac{2}{3}$	✓ S ✓ S (2)
9.1.2	$\frac{SV}{VR} = \frac{SW}{WP}$ [prop theorem/*eweredighst*; VW ‖ RP] $= \frac{2}{3}$	✓ answer (1)
9.2	$\frac{ST}{TQ} = \frac{SV}{VR}$ [both equal/*beide gelyk* $\frac{WS}{PW}$] $\therefore$ TV ‖ QR [line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*] $\therefore \hat{T}_1 = \hat{Q}_1$ [corresp/*ooreenkomst* ∠s/e; TV ‖ QR]	✓ S ✓ S ✓ R ✓ R (4)
9.3	ΔVWS ||| ΔRPS	✓ ΔRPS (any order) (1)
9.4	$\frac{WV}{PR} = \frac{SW}{SP}$ [ΔVWS ||| ΔRPS] $= \frac{2}{5}$ **OR/*OF*** $\frac{WV}{PR} = \frac{SV}{SR}$ [ΔVWS ||| ΔRPS] $= \frac{2}{5}$	✓ ratio ✓ answer (2) **[10]**

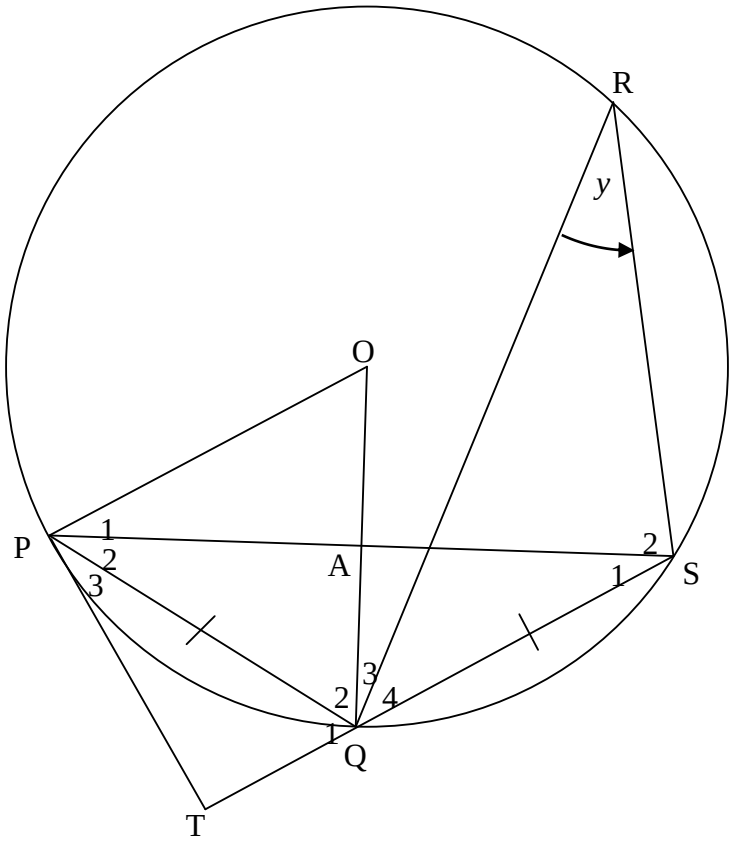
QUESTION/*VRAAG* 10

10.1



	Constr/*Konst* : Draw line PO and extend /*Trek lyn PO en verleng* Proof/*Bewys* : $OP = OA$ [radii] $\therefore \hat{P}_1 = \hat{A}$ [$\angle$s opp/*teenoor* = sides/*sye*] but $\hat{O}_1 = \hat{P}_1 + \hat{A}$ [ext $\angle$ of Δ] $\therefore \hat{O}_1 = 2\hat{P}_1$ Similarly/*Netso*, $\hat{O}_2 = 2\hat{P}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$ i.e. $A\hat{O}B = 2A\hat{P}B$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S (5)

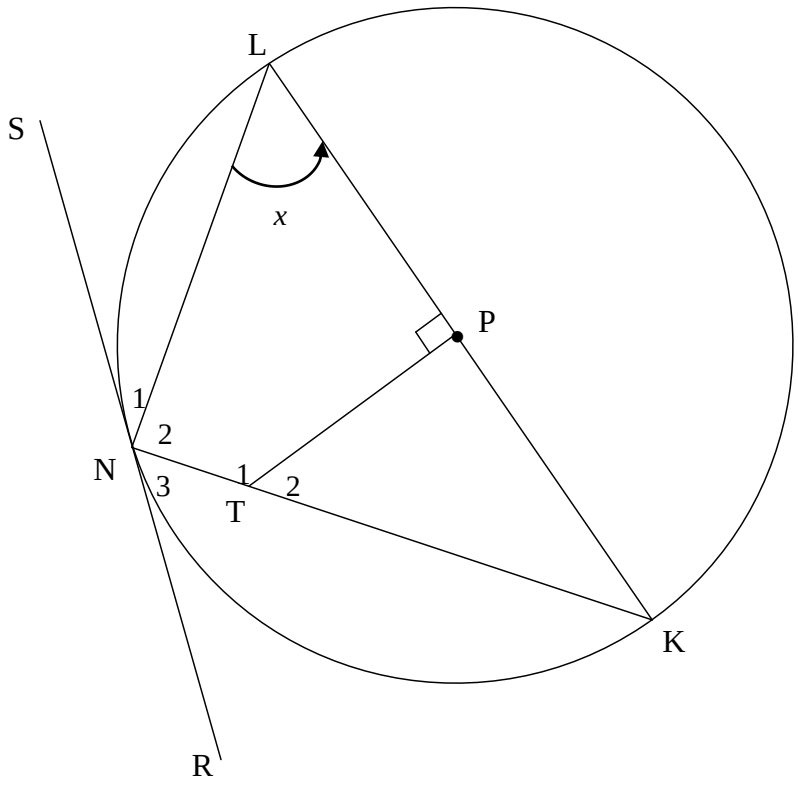
10.2



10.2.1	∠s in the same segment/ *∠e in dieselfde sirkelsegment*	✓ R (1)
10.2.2	$\hat{P}_2 = \hat{S}_1 = y$ [∠s opp equal sides/*∠e teenoor = sye*] $\hat{S}_1 = \hat{P}_3 = y$ [tan chord theorem/*raakl-koordst*] $\therefore\ \hat{P}_2 = \hat{P}_3$ $\therefore$ PQ bisects $T\hat{P}S$	✓ S ✓ R ✓ S ✓ R (4)
10.2.3	$P\hat{O}Q = 2\hat{S}_1 = 2y$ [∠at centre =2×∠at circ/*midpts∠= 2×omtreks∠]*	✓ S ✓ R (2)
10.2.4	$T\hat{P}A = \hat{P}_2 + \hat{P}_3 = 2y$ [proved/*bewys* in 11.2.2] $\therefore\ T\hat{P}A = P\hat{O}Q$ [proved/*bewys* in 11.2.3] $\therefore$ PT = tangent [converse tan chord theorem/*omgek raakl-koordst*]	✓ $T\hat{P}A = P\hat{O}Q$ ✓ R (2)

10.2.5	$O\hat{P}Q + O\hat{Q}P = 180° - 2y$ [sum of/*sum v ∠s/e* in Δ]	✓ S
	$\therefore O\hat{Q}P = 90° - y$ [∠s opp equal sides/*∠e to = sye;* OP = OQ]	✓ S ✓ R
	In ΔPAQ:	
	$O\hat{Q}P + \hat{P}_2 + Q\hat{A}P = 180°$	
	$90° - y + y + Q\hat{A}P = 180°$ [sum of/*sum v ∠s/e* in Δ]	✓ S
	$Q\hat{A}P = 90°$	✓ S
	$\therefore O\hat{A}P = 90°$ [∠s/*e* on straight line/*op reguitlyn*]	(5)
	OR/*OF*	
	$O\hat{P}T = 90°$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$\therefore \hat{P}_1 = 90° - 2y$	✓ S
	$\hat{P}_1 + \hat{O} + O\hat{A}P = 180°$ [sum of/*sum v ∠s/e* in Δ]	
	$(90° - 2y) + 2y + O\hat{A}P = 180°$	✓ S
	$\therefore O\hat{A}P = 90°$	✓ S (5)
	OR/*OF*	
	POSQ is a kite/'*n vlieër*	
	$\therefore OQ \perp PS$ [diag of a kite/*hoeklyne v vlieër*]	✓✓✓ S ✓✓ R
	$\therefore O\hat{A}P = 90°$	(5)
	OR/*OF*	
	In ΔOAP and ΔOAS	
	OP = OS (radii)	✓ S
	OA is common	✓ S
	$P\hat{O}A = 2y$	
	$= 2\hat{P}_2$	
	$= Q\hat{O}S$	✓ S
	$\Delta OAP \equiv \Delta OAS$ (SAS)	✓ R
	$O\hat{A}P = O\hat{A}S$ (≡Δs)	
	$O\hat{A}P = O\hat{A}S = 90°$ (∠s on str line)	✓ S
		(5) **[19]**

QUESTION/*VRAAG* 11



11.1	$\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*] ∴ TPLN is a cyclic quad/ 'n *koordevh* [opp ∠s of quad is suppl/ **t***eenoor∠e v vh is suppl*] **OR** $\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*] ∴ TPLN is a cyclic quad [ext ∠ = int opp ∠/*buite∠ = to binne ∠*]	✓ S ✓ R ✓ R (3) ✓ S ✓ R ✓ R (3)
11.2	$\hat{T}_2 = P\hat{L}N = x$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $\hat{K} = 90° - x$ [sum of/*som v* ∠s/e in Δ] $\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*] **OR/*OF*** $\hat{K} = 90° - x$ [sum of/*som v* ∠s/e in Δ] $\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*] **OR/*OF*** $\hat{N}_3 = x$ [tan chord theorem/*raakl-koordst*] $\hat{N}_2 = 90°$ [∠in semi circle/ *halfsirkel*] $\hat{N}_1 = 90° - x$ [straight line/*reguitlyn*]	✓ R ✓ S✓ R (3) ✓ R ✓ S✓ R (3) ✓ R ✓ S ✓ S (3)

11.3.1	In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\therefore \Delta KTP \mid\mid\mid \Delta KLN$ [$\angle\angle\angle$] **OR/*OF*** In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\hat{T}_2 = P\hat{L}N = x$ [proved in 11.2 **OR** sum of $\angle$s in Δ] $\therefore \Delta KTP \mid\mid\mid \Delta KLN$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
11.3.2	$\frac{KT}{KL} = \frac{KP}{KN}$ [$\mid\mid\mid$ Δs] $\therefore KT \cdot KN = KP \cdot KL$ But $KL = 2KP$ [radii: PK = LP] $\therefore KT \cdot KN = KP \cdot 2KP$ $= 2KP^2$ $= 2(KT^2 - TP^2)$ [Theorem of Pythagoras] $= 2KT^2 - 2TP^2$	✓ S/R ✓ S ✓ S ✓ S ✓ S (5) **[14]**

TOTAL/*TOTAAL*: 150