Arithmetic Sequences

An arithmetic sequence is a sequence in which there is an equal difference between each of the terms, i.e. we add/subtract the same number each time. For example:

\[ 2, \ 8, \ 14, \ 20, \ 26, \ 32... \]

In this sequence, we add 6 each time. This is our common difference, which we call \( d \). The first term, which we call \( a \), is 2.

In a more general form, using \( a \) and \( d \) in place of actual numbers, an arithmetic progression looks like this:

\[ a, \ a + d, \ a + 2d, \ a + 3d... \]

\( n^{th} \) term:

There is a general form for arithmetic progressions, which we can use to find the \( n^{th} \) term, that is: the term in a given position:

\[ T_n = a + (n - 1)d \]

Exercise 1:

Here are some examples using this formula:

1. Rufus gains 2 ears every day (starting with 2).
   a. How many ears will he have after a year? (365 days)
   b. After how many days will he have 22 ears?

2. Find the general form of the following arithmetic progressions:
   a. 18, 13, 8...
   b. \(-5, \ -8, \ -11...\)
   c. \(\frac{1}{2}, \ \frac{5}{2}, \ \frac{9}{2}...\)
   d. \(-10, \ -10, \ -10...\)
Sum:

We can also find the sum of an arithmetic progression up until a given term. While you could add up all the terms individually, this would take a very long time and isn't really a good idea. Instead, we use a formula.

Imagine being asked to add up all the numbers from 1 to 100. That is:

\[ 1, 2, 3, 4, 5, \ldots, 98, 99, 100 \]

This forms an arithmetic progression where \( a=1 \) and \( d=1 \). For interest, here is the general form of this progression:

\[
T_n = a + (n-1)d \\
T_n = 1 + (n-1) \times 1 \\
T_n = 1 + n - 1 \\
T_n = n + 1 - 1 \\
T_n = n
\]

A shortcut for finding the sum of this progression is this:

\[
\text{sum} = (\text{first term} + \text{last term}) \times (\text{number of pairs})
\]

In this example:

\[
\text{first term} = a = 1 \\
\text{last term} = T_n = a + (n-1)d = 100 \\
\text{number of pairs} = 100/2 = 50
\]

Therefore:

\[
\text{sum} = (1 + 100) \times 50 \\
\text{sum} = 101 \times 50 \\
\text{sum} = 5050
\]

We can use this to find the general formula for the sum of an arithmetic progression:

\[
S_n = \frac{n}{2}(a + \text{last term}) \\
S_n = \frac{n}{2}(a + T_n) \\
S_n = \frac{n}{2}(a + a + (n - 1)d)
\]
\[ S_n = \frac{n}{2}(2a + (n - 1)d) \]

**Exercise 2:**

Use the formula above to solve the following problems:

1. Find \( S_{20} \) (sum of 20 terms) of \( 4 + 10 + 16... \)
2. Find \( S_{15} \) of \( (-5) + (-8) + (-11)... \)
3. The sum of the first 4 terms of an arithmetic progression (\( S_4 \)) is equal to 26.
   - Find the **first term** if the common difference is 3.

Finding these easy enough? You can check your answers in our separate memo document.

Here's another exercise that puts everything we've covered together.

**Exercise 3:**

1. The 4th term of an arithmetic progression is 7. The 2nd term is \(-1\). Find the **first four terms** of the sequence.
2. The 5th term of an arithmetic progression is 60. The 4th is 50. What is the **first term**?
3. If \( T_3 = 16 \) and the first term is 8, **which term** (i.e. \( n = ? \)) is equal to 76?
4. Consider the sequence: 2, 5, 8...
   - a. Find \( S_{10} \)
   - b. **How many terms** will give a sum of 57?
5. Find the **first term** and **common difference** of an arithmetic progression where: \( T_3 = 13 \) and \( S_4 = 42 \)

Poor Rufus. I sincEARly hope he gets help.